New Tests of Expectation Formation with Applications to Asset Pricing Models*

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Abstract

We show unit root econometrics can be fruitfully employed to analyze expectations data and test expectation formation in financial and macroeconomic models with different informational assumptions. Survey data suggests stock price forecasts are not cointegrated with consumption forecasts and rejects this aspect of the formation of stock price expectations in a wide range of asset pricing models. The evidence casts some doubt on the modeling of expectation formation in asset pricing models which assume agents possess the knowledge of the equilibrium pricing function. Relaxing this knowledge appears necessary for models to reconcile the survey evidence and potential resolutions are discussed.

Keywords: Survey Expectation, Cointegration, Sentiment, Learning, Heterogeneous Beliefs

JEL classifications: D84, G12, G17.

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1 Introduction

Asset prices are crucially determined by investors' expectations about future. Yet asset pricing models are usually silent about to what extent model-implied asset price forecasts resemble forecasts made by agents in reality. Recent research employs survey expectations data to guide the modeling of expectation formation in financial markets and/or to discipline the modeling of asset price dynamics, examples are Greenwood and Shleifer (2014), Barberis, Greenwood, Jin and Shleifer (2015), Adam, Marcet and Beutel (2017), Adam, Matveev and Nagel (2018), and Nagel and Xu (2018).

Along this line, the paper develops new tests of expectation formation which are generally applicable in financial and macroeconomic models. A main contribution is enunciating the usefulness of unit root econometrics in analyzing expectations data and testing expectation formation in models with various informational assumptions.

We show these models typically impose a large amount of cointegration restrictions between forecasts of model variables and the tests utilize these restrictions. Researchers can apply these tests to study the cointegration between forecasts of exogenous variables and forecasts of endogenous variables in their model as well as the cointegration between forecasts of different endogenous variables. Moreover, these models impose cointegration restrictions between forecasts of the same variable (e.g., stock prices) over different forecasting horizons. Our tests make the most of survey expectations data as they can be applied even in cases when researchers only have limited data. For instance, forecasts of different variables in surveys may be made over different horizons or made at different dates. Or surveys only provide data on the forecast of the average value of economic variables over a number of periods (e.g., average GDP growth over next five years).

Asset pricing models typically imply stock price forecasts and aggregate consumption forecasts (or aggregate dividend forecasts) are cointegrated. While it would be interesting to study if stock price forecasts are cointegrated with dividend forecasts in the data, we do not pursue this analysis due to lack of high quality data on aggregate dividend forecasts with a sufficiently big sample size. A recent example is De La O and Myers (2018) which constructs analysts' forecasts of aggregate dividend index. Yet this data may be inappropriate for our purpose as their constructed data is relatively short (i.e., from 2003 to 2015).

A central new piece of evidence from expectations data uncovered by the paper is that median (or mean) survey stock price forecasts are not cointegrated with median (or mean) aggregate consumption forecasts. Put differently, the long-run or trend component of stock price forecasts is not anchored by consumption forecasts. This evidence is robust to different sources of expectations data, forecasting horizons, statistical tests, using median or mean forecasts for testing, and using stock price forecasts data which is made at different dates from consumption forecasts. The paper also finds that, for instance, forecasts of stock prices (or consumption) over different horizons in the data are cointegrated with each other, consistent with the stock pricing models we consider.

What does the survey evidence, especially the no cointegration between stock price forecasts and consumption forecasts, tell us about modeling expectation formation in asset pricing models? First, the evidence casts some doubt on the modeling of expectation formation in asset pricing models which assume the representative agent possesses the knowledge of the equilibrium pricing function, including (1) full-information RE models in an endowment or production economy setting, (e.g., Campbell and Cochrane (1999), Bansal, Kiku and Yaron (2012), Jermann (1998), Boldrin, Christiano and Fisher (2001), Croce (2014)), (2) models with learning consumption process (e.g., Collin-Dufresne, Johannes and Lochstoer (2016)) and (3) models with incorporating sentiment into consumption (e.g., Jin and Sui (2018)). Irrespective of agents having rational or non-rational (or extrapolative) expectations about consumption, stock price forecasts are cointegrated with aggregate consumption forecasts in these models, appearing inconsistent with the survey evidence. Intuitively, the long-run component of stock price forecasts in these models is anchored by consumption forecasts via agents' knowledge of the equilibrium pricing function.

Second, while large and persistent heterogeneity in beliefs about both expected stock returns and cash-flows among investors is a salient fact of the reality (see e.g., Giglio, Maggiori, Stroebel and Utkus (2019)), we show that in models with heterogeneous beliefs and a willingness to "agree to disagree" among agents (e.g., Ehling, Graniero, and Heyerdahl-Larsen (2018)), the average stock price forecasts across agents with different beliefs is cointegrated with the average aggregate consumption forecasts, inconsistent with our survey finding. Thus, our evidence casts some doubt on this type of models which also assumes investors have exact knowledge of the equilibrium pricing function and suggests a sufficiently large fraction of investors may not have this knowledge.[1]

[1]Strong informational assumptions on agents are usually required for them to derive the knowledge of

Third, the discrepancy between stationary price consumption ratios and nonstationary forecasts of price consumption ratios poses a challenge on asset pricing models which assume agents have the knowledge of the equilibrium pricing function. Reconciling the new survey evidence appears to require the assumption of agents' lack of knowledge of the equilibrium pricing function as in e.g., adaptive learning models; an example is Adam, Marcet and Beutel (2017, henceforth AMB). Yet relaxing this knowledge alone is insufficient for models to reproduce our survey finding because we show the typical specifications of stock price beliefs in adaptive learning models imply that stock price forecasts are cointegrated with consumption forecasts.

Fourth, the survey evidence suggests that major asset pricing models may have missed an important persistent component which independently drives stock price forecasts but not consumption forecasts. In a representative agent setting, one way to reconcile the evidence is modifying the belief specification in AMB by adding a non-stationary sentiment or judgment component (which is not cointegrated with consumption forecasts) *directly* to stock price forecasts together with assuming the agent's lack of knowledge of the equilibrium pricing function as in AMB.[2] Due to the agent's lack of this knowledge, this type of models can simultaneously produce stationary price to consumption (or dividend) ratios and nonstationary forecasts of price to consumption (or dividend) ratios. The key ingredients for models with heterogeneous beliefs to reproduce the evidence may be (1) a sufficiently large fraction of investors who do not possess the knowledge of the equilibrium pricing function and (2) a nonstationary sentiment or judgment variable which independently drives their stock price forecasts but not consumption forecasts.

Our tests can – but are not limited to – test the RE hypothesis. On the one hand, realized price consumption ratio is stationary.[3] On the other hand, we show forecasts of

the equilibrium pricing function in a wide range of asset pricing models, including full-information RE models, models with learning about consumption or dividend, sentiment-based models, and models with heterogeneous beliefs and a willingness to "agree to disagree". For instance, all agents' beliefs and preferences etc are common knowledge; see e.g., Adam and Marcet (2011).

[2] We also show how to empirically isolate this component of stock price forecasts and confirm it does not pass standard unit root tests and is not cointegrated with consumption forecasts data.

[3] Researchers who empirically test the stationarity of price consumption (or dividend) ratios appear to agree on this property which is a feature of almost all asset pricing models. And whether realized price consumption (or dividend) ratio is stationary or not does not affect our new survey evidence and our derived testable implications for the formation of stock price expectations in various full- and incomplete-information asset pricing models.

price consumption ratio is non-stationary. The discrepancy between realized and forecasts of price consumption ratio can be interpreted as a rejection of the RE hypothesis, in line with Greenwood and Shleifer (2014) and AMB which reject the RE hypothesis using stock market survey expectations data.

Survey data on expected stock returns are commonly criticized as being noisy and thus meaningless, or that people do not mean what they say, or that survey responses are strongly dependent on framing and language. Greenwood and Shleifer (2014) discusses and addresses these criticisms; see their Section 1.8. They show stock return forecasts from different surveys are highly correlated and provide evidence that investors act in line with their reported expectations. Giglio, Maggiori, Stroebel and Utkus (2019) also provides evidence addressing these criticisms and strongly supports the use of survey expectations data in macro-finance models. Moreover, we think, in our context, as long as the noises or measurement errors in survey forecast data are $i.i.d$ or stationary (which is commonly assumed in the literature), they do not affect the integration and cointegration properties of the forecast data as well as our empirical findings.

The paper relates to recent work on utilizing survey expectations data to test expectation formation and discipline financial modeling. Malmendier and Nagel (2011) show that investors' experience of macroeconomic outcomes affects their financial risk taking decisions. Greenwood and Shleifer (2014) and Adam, Marcet and Beutel (2017) both use survey stock return forecasts to reject the RE hypothesis. Adam, Matveev and Nagel (2018) empirically reject that survey return expectations are formed by risk-neutral investors or ambiguity averse/robust investors. Based on survey data findings (Malmendier and Nagel 2011, 2016), Nagel and Xu (2018) build an asset pricing model with learning with fading memory about dividend process to replicate several stock market facts such as counter-cyclical risk premium. Coibion and Gorodnichenko (2015) study the relation between ex post forecast errors and ex ante forecast revisions. They document widespread rejection of full-information RE models in the direction predicted by models of informational rigidities. Bordalo, Gennaioli, Ma and Shleifer (2018) find that forecasters typically over-react to their individual news, while consensus forecasts under-react to average news on individual stock earnings. To reconcile the findings, they combine a diagnostic expectation model of belief formation with a noisy information model of belief dispersion.

Section 2 develops new tests of expectation formation in full-information RE models.

New evidence on the formation of stock price expectations is provided in Section 3. Section 4 shows that stock price forecasts are cointegrated with consumption forecasts in major full-information RE asset pricing models. Cointegration tests of expectation formation are provided for models with incomplete information or sentiment (Section 5), models with heterogeneous beliefs (Section 6) and adaptive learning models (Section 7). Section 8 discusses potential resolutions for reconciling the new evidence. Section 9 concludes.

2 New tests of expectation formation in full-information RE models

This section develops new tests of expectation formation in full-information RE models (and later sections provide tests in several classes of incomplete-information RE models and non-RE models). These models impose a large amount of cointegration restrictions among forecasts of model variables and the tests utilize the restrictions. These tests can be generally applied by researchers to guide the modeling of expectation formation in financial and macroeconomic models. Our tests make the most of survey expectations data as they can be applied even if researchers have only limited data. For instance, forecasts of different variables in surveys may be made over different horizons or made at different dates. Or surveys only provide data on the forecast of the average value of model variables over a number of periods (e.g., average GDP growth over next five years).

Consider a variable $\{X_t\}$ from a full-information RE model, such as (log) stock price or aggregate consumption, which is generally represented by

$$X_t = X_t^P + X_t^C, \tag{1}$$

$$X_t^P = \mu + X_{t-1}^P + \sigma_{\epsilon,t}\epsilon_t, \tag{2}$$

$$(1-\phi(L))X_t^C = (1+\psi(L))\,\sigma_{\eta,t}\eta_t, \tag{3}$$

$$(1-\widetilde{\phi}(L))\left(\sigma_{\epsilon,t}^2 - \overline{\sigma}_\epsilon^2\right) = \left(1+\widetilde{\psi}(L)\right)\widetilde{\epsilon}_t, \tag{4}$$

$$(1-\widehat{\phi}(L))\left(\sigma_{\eta,t}^2 - \overline{\sigma}_\eta^2\right) = \left(1+\widehat{\psi}(L)\right)\widetilde{\eta}_t. \tag{5}$$

This variable contains a unit root. The superscripts P and C stand for the permanent and cyclical component. ϵ_t, η_t, $\widetilde{\epsilon}_t$ and $\widetilde{\eta}_t$ are $i.i.d$ innovations and independent to each other. The variance of ϵ_t and η_t are normalized to 1. $\sigma_{\epsilon,t}$ and $\sigma_{\eta,t}$ are allowed to be time-varying and their mean is constant and positive, i.e., $\overline{\sigma}_{\epsilon}^2$ and $\overline{\sigma}_{\eta}^2$. $\phi(L) = \phi_1 L + \phi_2 L^2 + ... + \phi_p L^p$ and $\psi(L) = \psi_1 L + \psi_2 L^2 + ... + \psi_q L^q$ where L is the lag operator. $\widetilde{\phi}(L)$, $\widetilde{\psi}(L)$, $\widehat{\phi}(L)$ and $\widehat{\psi}(L)$ are similarly defined.[4] The roots of $1 - \phi(z) = 0$, $1 - \widetilde{\phi}(z) = 0$, and $1 - \widehat{\phi}(z) = 0$ are within the unit circle, so X_t^C is a stationary process.

2.1 Integration property of conditional forecasts

Given the assumption of RE and full information, agents know the law of motion for X_t (equation (1)-(5)) and make use of this knowledge to make forecasts. The following lemma shows that if the variable X_t is integrated of order 1 ($X_t \sim I(1)$), conditional forecasts of this variable over arbitrary forecasting horizons i (i.e., $E_t X_{t+i}$) contain a unit root. For instance, if stock prices is an I(1) process, 1-year ahead forecasts of stock prices also contain a unit root.

Lemma 1 *If X_t follows (1)-(5) (i.e., $X_t \sim I(1)$), $E_t X_{t+i} \sim I(1)$ for $i > 0$.*

Proof. Given (1)-(5), we have $E_t X_{t+i} = E_t X_{t+i}^P + E_t X_{t+i}^C = \mu i + X_t^P + E_t X_{t+i}^C$. $E_t X_{t+i}$ is the sum of a unit root process and a stationary process and hence a unit root process. ■

2.2 Cointegration among forecasts of different variables

This section establishes the cointegration relationship among forecasts of different variables when their realizations are cointegrated. Researchers can apply these results to test the cointegration between forecasts of exogenous variables and forecasts of endogenous variables in their model. Moreover, they can study the cointegration between forecasts of different endogenous variables. Suppose $y_t = (y_{1,t}\ y_{2,t}\ ... y_{n,t})'$ is a $1 \times n$ vector which is cointegrated with cointegrating vector $a = (a_1\ a_2\ ...\ a_n)'$ and $a' y_t$ is a stationary process (with possibly time-varying volatility). Mathematically,

$$
\begin{aligned}
(1 - \phi(L)) a' y_t &= (1 + \psi(L))\, \sigma_{\eta,t} \eta_t, \\
(1 - \widehat{\phi}(L)) \left(\sigma_{\eta,t}^2 - \overline{\sigma}_{\eta}^2\right) &= \left(1 + \widehat{\psi}(L)\right) \widetilde{\eta}_t,
\end{aligned}
$$

[4] Specifically, $\widetilde{\phi}(L) = \widetilde{\phi}_1 L + \widetilde{\phi}_2 L^2 + ... + \widetilde{\phi}_{\widetilde{p}} L^{\widetilde{p}}$, $\widetilde{\psi}(L) = \widetilde{\psi}_1 L + \widetilde{\psi}_2 L^2 + ... + \widetilde{\psi}_{\widetilde{q}} L^{\widetilde{q}}$, $\widehat{\phi}(L) = \widehat{\phi}_1 L + \widehat{\phi}_2 L^2 + ... + \widehat{\phi}_{\widehat{p}} L^{\widehat{p}}$ and $\widehat{\psi}(L) = \widehat{\psi}_1 L + \widehat{\psi}_2 L^2 + ... + \widehat{\psi}_{\widehat{q}} L^{\widehat{q}}$.

where the roots of $1-\phi(z)=0$ and $1-\widehat{\phi}(z)=0$ are within the unit circle. We firstly establish a preliminary result which says the forecasts of an I(1) variable X made at date t over an arbitrary horizon i (i.e., E_tX_{t+i}) are cointegrated with X_k with cointegrating vector $(1,-1)$, where k can be identical to or different from t.

Lemma 2 *If X_t follows (1)-(5) (i.e., $X_t \sim I(1)$), $E_tX_{t+i}-X_k \sim I(0)$ for $i>0$.*

Proof. Let

$$\begin{aligned} E_tX_{t+i}-X_k &= \left(E_tX_{t+i}^P + E_tX_{t+i}^C\right) - X_t + (X_t - X_k) \\ &= \left(E_tX_{t+i}^P - X_t^P\right) + \left(E_tX_{t+i}^C - X_t^C\right) + (X_t - X_k) \\ &= \mu i + \left(E_tX_{t+i}^C - X_t^C\right) + (X_t - X_k)\,. \end{aligned}$$

$(E_tX_{t+i}-X_k)$ is stationary as $E_tX_{t+i}^C$, X_t^C and (X_t-X_k) are stationary. Note a special case is $t=k$. ■

Denote by $E_{i_1}y_{1,i_1+j_1}$ j_1-period ahead expectation of variable y_1 made at date i_1.

Theorem 3 *If $a'y_t$ is a stationary process, $a_1E_{i_1}y_{1,i_1+j_1}+a_2E_{i_2}y_{2,i_2+j_2}+...+a_nE_{i_n}y_{n,i_n+j_n}$ is stationary for arbitrary $i_1,i_2,...i_n,j_1,j_2,...,j_n>0$.*

Proof. Let

$$\begin{aligned} & \left[a_1E_{i_1}y_{1,i_1+j_1}+a_2E_{i_2}y_{2,i_2+j_2}+...+a_nE_{i_n}y_{n,i_n+j_n}\right] \\ = & \left[\sum_{k=1}^{n} a_k\left(E_{i_k}y_{k,i_k+j_k}-y_{k,i_k}\right)+\sum_{k=1}^{n} a_ky_{k,i_k}\right] \\ = & \left[\sum_{k=1}^{n} a_k\left(E_{i_k}y_{k,i_k+j_k}-y_{k,i_k}\right)+\sum_{k=1}^{n} a_ky_{k,i_1}+\sum_{k=2}^{n} a_k\left(y_{k,i_k}-y_{k,i_1}\right)\right]. \end{aligned}$$

Note Lemma 2 implies $(E_{i_k}y_{k,i_k+j_k}-y_{k,i_k})$ is stationary for $k=1,2,...n$. In addition, the cointegration of the vector y_t yields $\sum_{k=1}^{n} a_ky_{k,i_1}$ is stationary and $y_{k,t}\sim I(1)$ gives $(y_{k,i_k}-y_{k,i_1})$ is stationary. Thus, we have $a_1E_{i_1}y_{1,i_1+j_1}+a_2E_{i_2}y_{2,i_2+j_2}+...+a_nE_{i_n}y_{n,i_n+j_n}$ is stationary. ■

The theorem contains a rich set of testable implications for expectation formation. For illustration, consider the asset pricing models discussed later (e.g., the long-run risks model and habit model) in which realized stock prices and consumption are cointegrated with cointegrating vector $(1, -1)$. First, a special case of the theorem is that forecasts of stock prices and consumption made at the same date (i.e., $i_1 = i_2 = ... = i_n$) and over the same forecasting horizons (i.e., $j_1 = j_2 = ... = j_n$) are cointegrated. And forecasts of stock price consumption ratio, i.e., $(E_t \log P_{t+j} - E_t \log C_{t+j})$ is stationary. This means, for example, 1-year ahead forecasts of stock prices and 1-year ahead forecasts of consumption (made at the same date) are cointegrated with cointegrating vector $(1, -1)$.

Second, the cointegration relation holds for forecasts of different variables *over different forecasting horizons* (i.e., $j's$ need not to be identical) as $(E_t \log P_{t+j_1} - E_t \log C_{t+j_2})$ is stationary for $j_1 \neq j_2$. This means, for instance, 10-year ahead forecast of stock prices and 1-year ahead forecast of consumption made at the same date are cointegrated. This result is particularly useful when the forecasting horizons of expectation data available to researchers are different across different variables. For instance, researchers may have data on 10-year ahead forecasts of stock prices and 1-year ahead (but not 10-year ahead) forecasts of consumption.

Third, the cointegration relation also holds for forecasts of different variables *made at different dates* (i.e., $i's$ need not to be identical) as $(E_{i_1} \log P_{i_1+j1} - E_{i_2} \log C_{i_2+j_2})$ is stationary for $i_1 \neq i_2$. This means, for instance, stock price forecasts made during $1960 - 1990$ (over an arbitrary forecasting horizon) are cointegrated with consumption forecasts made during $1970 - 2000$ (over an arbitrary forecasting horizon). This result is useful when the sample period of expectation data available to researchers is different (or do not exactly overlap) across different variables.

Perhaps surprisingly, all testable implications (i.e. cointegration restrictions) are also present in various learning and sentiment-based models, as is shown later.

2.3 Cointegration among forecasts of the same variable

The following theorem shows that the forecasts of the same I(1) variable made at the same date i over two arbitrary and different horizons $j \neq l$ are cointegrated with cointegrating vector $(1 \ -1)$. This means, for instance, 1-year ahead and 10-year ahead forecasts of stock prices made at the same date are cointegrated. In addition, the forecasts of the same variable made at two different dates (i.e., $i \neq k$) over two arbitrary horizons ($i.e., j$ and l) are cointegrated with cointegrating vector $(1 \ \ -1)$.

Theorem 4 *If X_t follows (1)-(5) (i.e., $X_t \sim I(1)$), $E_iX_{i+j} - E_kX_{k+l} \sim I(0)$ for (a) $i = k$, $j \neq l$ or (b) $i \neq k$, $j > 0$, $l > 0$.*[5]

Proof. First, consider case (a) when $i = k$ and $j \neq l$. Let $E_iX_{i+j} - E_iX_{i+l} = (\mu j + X_i^P + E_iX_{i+j}^C) - \left(\mu l + X_i^P + E_iX_{i+l}^C\right) = \mu(j-l) + \left(E_iX_{i+j}^C - E_iX_{i+l}^C\right)$. $(E_iX_{i+j} - E_iX_{i+l})$ is stationary because $(E_iX_{i+j}^C - E_iX_{i+l}^C)$ is stationary. Turning to case (b) when $i \neq k$. Let $E_iX_{i+j} - E_kX_{k+l} = (E_iX_{i+j} - X_i) - (E_kX_{k+l} - X_k) + (X_i - X_k)$. Lemma 2 yields that $(E_iX_{i+j} - X_i)$ and $(E_kX_{k+l} - X_k)$ are stationary. Moreover, given $X_t \sim I(1)$, $(X_i - X_k)$ is stationary. Thus, $(E_iX_{i+j} - E_kX_{k+l})$ is stationary. ■

2.4 Tests using average forecasts over many periods

Economic surveys often ask participants their forecast of the average value of economic variables X_t over the next m periods, for instance, average unemployment rate over next five years. This section provides testable implications for average forecasts when $X_t \sim I(1)$. The tests are useful when researchers have data on average expectations over a number of periods.[6]

Define the average forecast $\overline{X}_t^m = \frac{1}{m}\sum_{i=1}^{m} E_tX_{t+i}$ where the average is calculated over a number of time periods (rather than across different survey participants). Part (1) of the following Lemma shows that the average forecasts $\overline{X}_t^m$ contain a unit root. Part (2) shows that the average forecasts of X over the next m periods made at an arbitrary date h $(\overline{X}_t^m)$ are cointegrated with conditional forecasts of X over horizon l made at an arbitrary date j (E_jX_{j+l}) with cointegrating vector $(1, -1)$. Part (3) shows that $\overline{X}_t^m - X_j \sim I(0)$.

Lemma 5 *If X_t follows (1)-(5) (i.e., $X_t \sim I(1)$), then (1) $\overline{X}_t^m \sim I(1)$ for $m > 0$; (2) $\overline{X}_h^m - E_jX_{j+l} \sim I(0)$ for arbitrary h, j, m and $l > 0$; (3) $\overline{X}_t^m - X_j \sim I(0)$ for arbitrary t, j, $m > 0$.*

[5] Note if $i = k$ and $j = l$, E_iX_{i+j} and E_kX_{k+l} are identical to each other.

[6] Note they are not used in empirical testing of the paper as we do not have average expectations data in the current context.

Proof. (1) Let $\overline{X}_t^m - \overline{X}_{t-1}^m = \frac{1}{m}\sum_{i=1}^{m}(E_tX_{t+i} - E_{t-1}X_{t-1+i})$. Lemma 1 implies that $E_tX_{t+i} \sim I(1)$ and hence $(E_tX_{t+i} - E_{t-1}X_{t-1+i})$ is stationary. Thus, $\left(\overline{X}_t^m - \overline{X}_{t-1}^m\right)$ is stationary.

(2) For arbitrary h, j, m and l, let $\overline{X}_h^m - E_jX_{j+l} = \frac{1}{m}\sum_{i=1}^{m} E_hX_{h+i} - E_jX_{j+l} = \frac{1}{m}\sum_{i=1}^{m}(E_hX_{h+i} - E_jX_{j+l})$. Theorem 4 shows that $(E_hX_{h+i} - E_jX_{j+l})$ is stationary. Thus, $\overline{X}_h^m - E_jX_{j+l}$ is stationary.

(3) Let $\overline{X}_t^m - X_j = \frac{1}{m}\sum_{i=1}^{m} E_tX_{t+i} - X_j = \frac{1}{m}\sum_{i=1}^{m}(E_tX_{t+i} - X_j)$. Lemma 2 implies that $(E_tX_{t+i} - X_j)$ is stationary. Thus, we have $\left(\overline{X}_t^m - X_j\right)$ is stationary. ■

Let $y_t = (y_{1,t}\ y_{2,t}\ ...y_{n,t})'$ be a $1 \times n$ vector which is cointegrated with cointegrating vector $a = (a_1\ a_2\ ...\ a_n)'$ and denote $Z_{k,i_k}^{j_k} = E_{i_k}y_{k,i_k+j_k}$ or $\overline{y}_{k,i_k}^{j_k}$ where $\overline{y}_{k,i_k}^{j_k} = \frac{1}{j_k}\sum_{l=1}^{j_k} E_{i_k}y_{k,i_k+l}$.

Theorem 6 *$a_1Z_{1,i_1}^{j_1} + a_2Z_{2,i_2}^{j_2} + ... + a_nZ_{n,i_n}^{j_n}$ is stationary for arbitrary $i_1, i_2, ...i_n$, j_1, j_2, ..., $j_n > 0$.*

Proof. Let $a_1Z_{1,i_1}^{j_1} + a_2Z_{2,i_2}^{j_2} + ... + a_nZ_{n,i_n}^{j_n} = \left[\sum_{k=1}^{n} a_k\left(Z_{k,i_k}^{j_k} - y_{k,i_k}\right) + \sum_{k=1}^{n} a_ky_{k,i_k}\right]$. It is stationary for two reasons. First, Lemma 2 and part (3) of Lemma 5 imply that $\left(Z_{k,i_k}^{j_k} - y_{k,i_k}\right)$ is stationary for $Z_{k,i_k}^{j_k} = E_{i_k}y_{k,i_k+j_k}$ or $\overline{y}_{k,i_k}^{j_k}$. Second, $\sum_{k=1}^{n} a_ky_{k,i_k}$ is stationary as is shown in the proof of Theorem 3. ■

Theorem 6 shows that if a vector of variables are cointegrated, a linear combination of the average and conditional forecasts of these variables (with the same cointegrating vector) is stationary. Note a special case is when the forecast of all variables are made at the same date ($i_1 = i_2 = ... = i_n$).

3 New evidence on the formation of stock price expectations

Using the tests developed from previous section, this section presents new evidence on the formation of stock price expectations. A central piece of evidence from expectation data is that stock price forecasts are not cointegrated with consumption forecasts, as opposed to full-information RE asset pricing models considered in the next section (e.g., the long-run risks model in Bansal, Kiku and Yaron, 2012 or the habit model of Campbell and Cochrane, 1999).[7] Put differently, the long-run or trend component of stock price forecasts is *not* anchored by consumption forecasts. This evidence is robust to different sources of expectations data, forecasting horizons, statistical tests, using median or mean forecasts for testing, and using stock price forecasts data which is made at different dates from consumption forecasts. In subsequent sections, we show several classes of asset pricing models with incomplete information or non-RE appear inconsistent with this evidence too.

3.1 Data

Two sources of survey forecasts of US stock prices are used. One is the Livingston Survey managed by the Federal Reserve Bank of Philadelphia. The survey contains forecasts of S&P 500 index made by professional economists from industry, government, banking and academia. The stock price forecast data is semi-annual and covers from 1952 to the second half of 2017.[8] Two forecasting horizons are available: 2- and 4-quarter ahead. The other source is Robert Shiller's survey of individual investors. This forecast of stock prices is measured by forecasts of the Dow Jones index and available at quarterly frequency. The data covers from the first quarter of 1999 to the second quarter of 2015. Four forecasting horizons are available: 1-quarter, 2-quarter, 4-quarter and 10-year ahead. Both survey forecasts of stock prices are deflated by forecasts of inflation rate obtained from the Survey of Professional Forecasters (SPF) conducted by the Philadelphia Fed. The forecasting horizons of inflation forecast data are 1- to 4- quarter ahead as well as 10-year ahead.[9]

Two sources of US aggregate consumption forecasts are used. One is SPF forecasts of the chain-weighted real personal consumption expenditures. It is available at quarterly frequency and from 1981 Q3 onwards. SPF consumption forecasts data is provided with

[7] Other evidence includes that forecasts of stock prices (or consumption) over different forecasting horizons are cointegrated with each other, consistent with all asset pricing models considered in the paper.

[8] In all cases (with one exception, i.e., Table 4), we use the data from 1981 onwards which corresponds to the longest sample of consumption forecasts in the Survey of Professional Forecasters.

[9] For robustness analysis, we also deflate 1-year ahead stock price forecasts using 1-year ahead inflation forecasts using the Michigan Survey of Consumers. Our results are robust to this alternative measure of inflation expectation.

varying base years. Appendix A explains the rebasing of consumption forecast data. As an alternative, consumption forecasts from the US Federal Reserve Board's Greenbook datasets are employed. In the text, we report testing results using SPF consumption forecasts. Most results reported in the text use median survey forecasts. Appendix B (Appendix C) shows our results are robust to using mean forecasts (Greenbook consumption forecasts). Figure 1 plots the (normalized) median forecasts of (log) stock prices and rebased aggregate consumption for all available forecasting horizons.

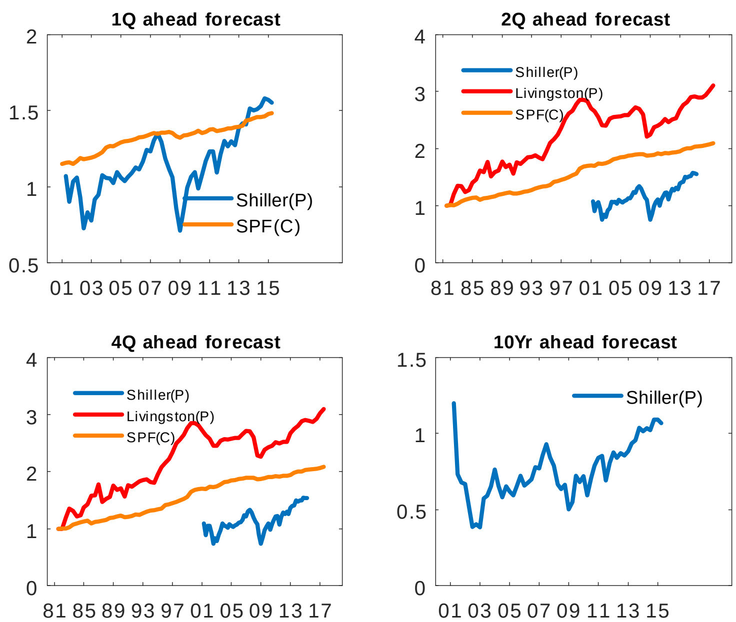

Figure 1: Median forecasts of (log) stock price and consumption

Before proceeding to the test results, we discuss several issues. First, it is pervasive in the literature to use mean or median forecasts from surveys of expectations (e.g., SPF, Livingston, Shiller) as a proxy for the expectations of the representative agent, such as Piazzesi, Salomao and Schneider (2015), and Coibion and Gorodnichenko (2015). With this view, it may be appropriate for us to use stock price forecasts from one survey and consumption forecasts from another survey to test the cointegration between the forecasts of the two variables made by the representative agent in asset pricing models, despite that the set of forecasters are not the same in different surveys.[10] Most papers in the literature

[10] Carroll (2003) develops a model of expectation formation in which households derive their expectations from professional forecasters and provides evidence supporting this model using inflation expectations data.

similarly follow this practice. For instance, Adam, Marcet and Beutel (2017) uses median (or mean) nominal stock price forecasts from Shiller Survey and inflation forecasts from SPF and Michigan Survey of Consumers to compute median (or mean) forecasts of real stock capital gains which is then used as a proxy for forecasts of real stock capital gains made by the representative agent in their model. Another example is Malmendier and Nagel (2011) which documents heterogeneity in inflation expectations using Michigan Survey of Consumers and then studies empirically the implications of this heterogeneity for borrowing and lending decisions using data from the Survey of Consumer Finances. Second, Section 6 provides tests of expectation formation in asset pricing models with heterogeneous agents and beliefs. For testing, the mean forecasts across agents with different beliefs in these models are proxied by mean survey forecasts.

Third, survey data on expected return or stock prices are commonly criticized (1) as being noisy and thus meaningless and (2) that people do not mean what they say or that survey responses are strongly dependent on framing and language. Greenwood and Shleifer (2014) and Giglio, Maggiori, Stroebel and Utkus (2019) address these criticisms and strongly support the use of survey expectation data; see the discussion in the Introduction. We think, moreover, if the noises or measurement errors in survey stock price and consumption forecasts are *i.i.d* over time or follow a stationary process, all our testing results below are not affected because they do not change the integration and cointegration properties of the forecasts data.

Fourth, the power of standard Dickey-Fuller class of unit root tests were frequently criticized in 1980s and 1990s. Subsequent work has made great advances in improving the power of the tests. Ng and Perron (2001) and Haldrup and Jansson (2006) argue some subsequently developed tests have much improved or excellent power. The paper uses some of the most powerful tests like the DF-GLS test. Another way to alleviate the problem is applying the KPSS test which tests the null hypothesis of a stationary process against the alternative of a unit root. Our conclusion still holds with this test even after we take account of potential small sample problems using Monte Carlo simulation; see Appendix D.[11] For instance, we

If this model is generally true (for other variables), forecasts of a variable (e.g., stock prices or consumption) made by professional forecasters will be cointegrated with forecasts of the same variable made by households. This may also help to justify our use of different surveys where forecasts of stock prices and consumption are not made by the same set of forecasters.

[11]One might still think failing to reject the cointegration between stock price forecasts and consumption forecasts does not necessarily mean forecasts of stock price consumption ratio is non-stationary. In this case, our tests can be interpreted as providing test statistics which can be used to discipline the modeling of asset

simulate the distribution of the KPSS test statistic under the null hypothesis that the long-run risks model of Bansal, Kiku, and Yaron (2012) is true (and hence forecasts of $log(P/C)$ over different horizons are stationary). Survey forecasts data rejects this null hypothesis using the critical values obtained from the Monte Carlo study.

3.2 Integration properties of the forecasts

Table 1: Integration properties: forecasts of $logP$

	1Q ahead	2Q ahead	4Q ahead	10-yr ahead
Panel A: I(1) test				
Shiller (PP Z_t stat.)	-2.231	-2.183	-2.242	-1.997
10% critical value	-3.172	-3.172	-3.172	-3.172
Shiller (DF-GLS)	-2.236	-2.150	-2.183	-1.317
10% critical value	-2.851	-2.851	-2.851	-2.851
Livingston (PP Z_t stat.)	$n.a.$	-2.195	-2.118	$n.a.$
10% critical value	$n.a.$	-3.167	-3.167	$n.a.$
Livingston (DF-GLS)	$n.a.$	-1.714	-1.756	$n.a.$
10% critical value	$n.a.$	-2.818	-2.818	$n.a.$
Panel B: I(2) test				
Shiller (PP Z_t stat.)	-7.618	-7.596	-8.012	-9.351
1% critical value	-2.615	-2.615	-2.615	-2.615
Shiller (DF-GLS)	-3.607	-3.505	-3.311	-1.518
1% critical value	-2.615	-2.615	-2.615	-2.615
Livingston (PP Z_t stat.)	$n.a.$	-7.206	-6.950	$n.a.$
1% critical value	$n.a.$	-2.612	-2.612	$n.a.$
Livingston (DF-GLS)	$n.a.$	-4.528	-3.084	$n.a.$
1% critical value	$n.a.$	-2.611	-2.611	$n.a.$

This section studies the integration properties of forecasts of aggregate stock price index and aggregate consumption. Table 1 reports the test statistics and critical value of the Phillips-Perron (PP) test (see Phillips and Perron (1988)) and the Augmented Dickey-Fuller Generalized Least Squares (DF-GLS) test for median forecasts of stock prices. Panel A

prices.

shows that for both surveys and all forecast horizons, both tests cannot reject that stock price forecasts is I(1) at 10% significance level.[12] Panel B shows that for both surveys and all forecasting horizons (with one exception), stock price forecasts is not integrated of order 2, i.e., I(2).[13] Table 2 reports the test statistic value and critical value of the unit root tests for aggregate consumption forecasts. Similarly, for all forecasting horizons, both tests suggest that consumption forecasts is an I(1) but not I(2) process. Lemma 1 suggests that forecasts of stock prices and consumption from full-information RE stock pricing models (e.g., the long-run risks model) are I(1) but not I(2) processes, consistent with the evidence here.

Table 2: Integration properties: forecasts of $logC$ (SPF)

	1Q ahead	2Q ahead	4Q ahead
Panel A: I(1) test			
PP (Z_t stat.)	−1.324	−1.327	−1.340
10% critical value	−3.167	−3.167	−3.167
DF-GLS	−1.223	−1.227	−1.238
10% critical value	−2.818	−2.818	−2.818
Panel B: I(2) test			
PP (Z_t stat.)	−4.725	−4.805	−4.837
1% critical value	−2.612	−2.612	−2.612
DF-GLS	−3.809	−4.082	−3.862
1% critical value	−2.611	−2.611	−2.611

3.3 No cointegration between stock price forecasts and consumption forecasts

Figure 2 displays the difference between median forecasts of $logP$ and $logC$ made at the same date using both stock price surveys. "1Q ahead forecast" corresponds to (normalized) 1-year ahead stock price forecasts minus 1-year ahead SPF consumption forecasts; similarly

[12] DF-GLS test gives all test statistics for a series of models that include 1 to k lags of the first differenced, detrended variable, where k is set by default. We report the statistics produced with the number of lags leading to the lowest mean squared errors. And the results are robust to alternative lags.

[13] The only exception is the DF-GLS test cannot reject that 10-year ahead median forecast of stock prices from the Shiller Survey follows an I(2) process. Yet we show that it is rejected using the mean forecast at 1% significance level, see column 4 of Table A1 in Appendix B.

for 2Q and 4Q ahead forecast. The exception is "10 Yr ahead forecast" which corresponds to 10-year ahead stock price forecasts minus 1-year ahead consumption forecasts, given the unavailability of 10-year ahead consumption forecasts in the SPF.

Recall Theorem 3 implies that in full-information RE asset pricing models, stock prices forecasts and consumption forecasts made at the same dates (and over possibly different horizons) are cointegrated with cointegrating vector $(1, -1)$. These models imply, for instance, 1-quarter ahead forecasts of stock prices are cointegrated with 1-quarter ahead forecast of aggregate consumption and 10-year ahead forecast of stock prices are cointegrated with 1-year ahead forecast of consumption.

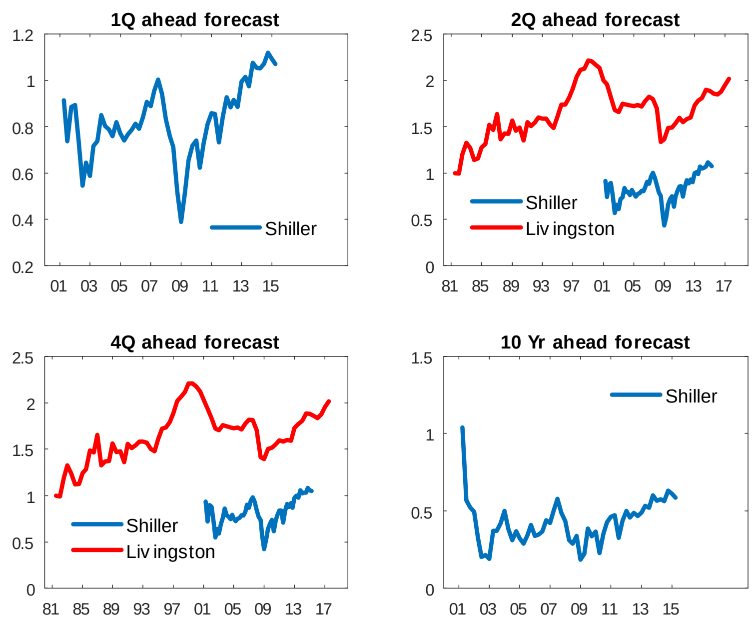

Figure 2: Median forecast of (log) price consumption ratio

Note: the bottom right panel plots the (normalized) difference between 10-year ahead stock price forecasts and 1-year ahead consumption forecasts.

Table 3 reports the test results of whether median forecasts of aggregate consumption are cointegrated with median forecasts of stock prices made at the same date and over the same forecasting horizon (with cointegrating vector $(1, -1)$);[14] the only exception is that the

[14]For DF-GLS test, we report the test statistics produced with the number of lags leading to the lowest mean squared errors. The results are robust to different choices of lags.

column "10-yr ahead" is the test results on the cointegration between 10-year ahead forecasts of stock prices and 1-year ahead forecasts of consumption, given the unavailability of 10-year ahead consumption forecasts data. Both PP and DF-GLS tests show that we cannot reject the null hypothesis that stock price forecasts are not cointegrated with consumption forecasts with cointegrating vector $(1, -1)$, robust to different data sources and forecasting horizons. The same conclusion is reached with mean forecasts, see Table A3 of the Appendix. We also applied the KPSS test which tests the null hypothesis of a stationary process against the alternative of a unit root. The tests yield the same conclusion that forecasts of stock price to consumption ratio is non-stationary, robust to different sources of forecasts data, different forecasting horizons, correcting potential small sample problems etc (see Appendix D).

Table 3: No cointegration between forecasts of $\log P$ and $\log C$

	1Q ahead	2Q ahead	4Q ahead	10-yr ahead*
I(1) test				
Shiller (PP Z_t stat.)	−2.039	−1.962	−2.097	−2.304
10% critical value	−2.595	−2.595	−2.595	−2.595
Shiller (DF-GLS)	−1.665	−1.569	−1.602	−0.814
10% critical value	−1.929	−1.929	−1.929	−1.929
Livingston (PP Z_t stat.)	*n.a.*	−2.234	−2.161	*n.a.*
10% critical value	*n.a.*	−2.591	−2.591	*n.a.*
Livingston (DF-GLS)	*n.a.*	−0.246	−0.227	*n.a.*
10% critical value	*n.a.*	−1.895	−1.895	*n.a.*

*Note: for the first three columns, forecasts of stock prices and consumption are made at the same date and over the same forecasting horizons. For the fourth column, the forecasts of stock prices and consumption are made at the same dates but different forecasting horizons (i.e.,10-year ahead stock price forecasts vs 1-year ahead consumption forecasts).

Theorem 3 also suggests stock price forecasts and consumption forecasts *made at different dates* are cointegrated with cointegrating vector $(1, -1)$. Denote by $E_{i_1} \log P_{i_1+j_1}$ stock price forecasts made at period i_1 over horizon j_1 and similarly $E_{i_2} \log C_{i_2+j_2}$ consumption forecasts made at period i_2 over horizon j_2. A period means a quarter here. For illustration, we conduct two exercises. In the first exercise, we use 1-year ahead Livingston median stock price forecasts ($j_1 = 4$) where i_1 = 1978Q4, 1979Q2, 1979Q4, ..., 2014Q2. And we take

1-year ahead SPF median consumption forecasts ($j_2 = 4$) where $i_2 = 1981Q4$, 1982Q2, 1982Q4, ..., 2017Q4.[15] Note that stock price forecasts and consumption forecasts are made at different dates, i.e., the set of i_1 is different from the set of i_2. Yet the sample size of stock price forecasts and consumption forecasts are the same. In the second exercise, we use 1-year ahead Livingston median stock price forecasts ($j_1 = 4$) where $i_1 = 1978Q2$, 1978Q4, 1979Q2, ...2017Q4 and 1-year ahead SPF median consumption forecasts ($j_2 = 4$) where $i_2 = 1998Q2$, 1998Q3, 1998Q4, ..., 2018Q1. Using PP and DF-GLS tests, Panel A (or B) of Table 4 reports the testing results of the first (or second) exercise. Both panels suggest no cointegration between stock price forecasts and consumption forecasts made at different dates.[16]

Table 4: Testing cointegration between $E_{i_1} \log P_{i_1+j_1}$ & $E_{i_2} \log C_{i_2+j_2}$

Panel A: $i_1 = 78Q4, 79Q2, ..., 14Q2$ and $j_1 = 4$; $i_2 = 81Q4, 82Q2, ..., 17Q4$ and $j_2 = 4$		
	Median forecasts	Mean forecasts
I(1) test		
PP (Z_t statistics)	-1.628	-1.603
10% critical value	-2.591	-2.591
DF-GLS	-1.080	-0.964
10% critical value	-1.895	-1.895
Panel B: $i_1 = 78Q2, 78Q4, ..., 17Q4$ and $j_1 = 4$; $i_2 = 98Q2, 98Q3, ..., 18Q1$ and $j_2 = 4$		
	Median forecasts	Mean forecasts
PP (Z_t stat.)	-1.501	-0.680
10% critical value	-2.588	-2.588
DF-GLS	-0.167	-0.137
10% critical value	-1.871	-1.825

[15] Note the Livingston survey is a semi-annual survey and conducted close to the end of June and December each year. Since SPF is a quarterly survey, some data is discarded for the test, i.e., those made in the first and third quarter of each year.

[16] Of course there are many possible ways of utilizing the survey expectations data. One may choose different sets of i_1 and i_2 from our choice here as long as the two forecasts have the same sample size. We find other choices also lead to the same conclusion but it may be impractical and unnecessary to report all results here.

The long-run or trend component of stock price forecasts made by agents in reality is not anchored by consumption forecasts. The survey evidence rejects this aspect of the formation of stock price expectations in full-information RE asset pricing models and in various learning or sentiment-based models, as is shown later.

We also test if stock price forecasts are cointegrated with consumption forecasts without imposing the restriction of cointegrating vector $(1, -1)$. Using the Livingston Survey data, we find that the null hypothesis that stock price forecasts are not cointegrated with consumption forecasts for any forecasting horizons cannot be rejected at conventional significance level (e.g., 5%). Using the shorter Shiller Survey data, we reject the null hypothesis that stock price forecasts are not cointegrated with consumption forecasts at 5% significance level. The estimated cointegrating vector is (1, -2.55) for 1-year ahead forecasts of stock prices and consumption; similar cointegrating vectors are found for other forecasting horizons. This also rejects the formation of stock price expectations in the asset pricing models studied in the paper which has $(1, -1)$ as the cointegration vector.

3.4 Cointegration among stock price (or consumption) forecasts

In full-information RE models (e.g., the long-run risks model), realized stock prices (or consumption) contains a unit root. Theorem 4 implies that forecasts of stock prices (or consumption) over two different horizons should be cointegrated with cointegrating vector $(1, -1)$. This section shows that this aspect of expectation formation in these models is broadly consistent with survey forecasts of stock prices and consumption.

Table 5: P-value of testing the stationarity of $E_t \log(X_{t+i}) - E_t \log(X_{t+j})$ (X stands for stock prices or consumption)

PP test	1 & 2Q	1 & 4Q	2 & 4Q
Stock prices (Shiller median)	0.0019	0.0005	0.0047
Stock prices (Shiller mean)	0.0019	0.0005	0.0047
Consumption (SPF median)	0.0004	0.0228	0.0011
Consumption (SPF mean)	0.0006	0.0035	0.0000

Table 5 reports the p-value of PP test on whether forecasts of stock prices (or aggregate consumption) over two different horizons are cointegrated with each other with cointegrating vector $(1, -1)$. Using median and mean forecasts data, PP test shows that at 1% significance level, we can reject the null hypothesis that the difference between the forecasts of stock prices, i.e., $E_t \log(p_{t+i}) - E_t \log(p_{t+j})$, contains a unit root, for various pairs of forecasting horizons $(i, j) = (1, 2)$, $(i, j) = (1, 4)$ and $(i, j) = (2, 4)$.[17] Similarly, forecasts of consumption over two different horizons are cointegrated with cointegrating vector $(1, -1)$.

What does the survey evidence, especially the no cointegration between stock price forecasts and consumption forecasts, tell us about modeling expectation formation in asset pricing models? In the next four sections, we study if the new survey evidence can be reconciled in models with rational expectations, or non-rational (or extrapolative) consumption forecasts, or deviation from the representative agent assumption, or non-rational stock price forecasts.

4 Full-information RE asset pricing models

This section demonstrates that stock prices and aggregate consumption are cointegrated in several major (endowment or production economy) full-information RE asset pricing models. The tests in Section 2 can be applied. Particularly, Theorem 3 implies that stock price forecasts are cointegrated with aggregate consumption forecasts in these models. Thus, the survey evidence in Section 3.3 appears incompatible with these models.

4.1 The long-run risks model

Consider the long-run risks model studied in Bansal, Kiku and Yaron (2012). The representative agent with recursive preferences maximizes his life-time utility given by

$$V_t = [(1 - \delta) C_t^{\frac{1-\gamma}{\theta}} + \delta (E_t[V_{t+1}^{1-\gamma}])^{\frac{1}{\theta}}]^{\frac{\theta}{1-\gamma}}. \tag{6}$$

The variable θ is defined as $\theta \equiv \frac{1-\gamma}{1-1/\psi}$ where the parameters γ and ψ represent relative risk aversion and the elasticity of intertemporal substitution. Log consumption c_t and dividend

[17] DF-GLS test rejects the null hypothesis that aggregate price index forecasts across different horizons are not cointegrated at 1 percent level. It also accepts the cointegration between 1 quarter ahead and 2 quarter ahead consumption forecast. However it fails to reject that 1- (or 2-) quarter ahead consumption forecast is not cointegrated with 4 quarter ahead consumption forecast.

d_t have the following joint dynamics

$$\Delta c_{t+1} = \mu_c + x_t + \sigma_t \eta_{t+1}, \quad (7)$$
$$x_{t+1} = \rho x_t + \varphi_e \sigma_t e_{t+1}, \quad (8)$$
$$\sigma^2_{t+1} = \overline{\sigma}^2 + \nu(\sigma^2_t - \overline{\sigma}^2) + \sigma_w w_{t+1}, \quad (9)$$
$$\Delta d_{t+1} = \mu_d + \phi x_t + \pi \sigma_t \eta_{t+1} + \varphi \sigma_t u_{d,t+1}. \quad (10)$$

$\mu_c + x_t$ is the conditional expectation of the growth rate of aggregate consumption. x_t is a persistent component which captures long run risks in consumption and drives both the consumption and dividend process. ϕ captures a levered exposure of dividend to x_t. In addition, the i.i.d consumption shock η_{t+1} is allowed to influence the dividend process. It serves as an additional source of risk premia and π governs the magnitude of this influence.

Their paper provides the analytical solution for (log) price-consumption ratio

$$\log(\frac{P_t}{C_t}) = A_0 + A_1 x_t + A_2 \sigma^2_t, \quad (11)$$

where A_0, A_1, A_2 are all constants and functions of model parameters, see their p. 189. Stock prices and aggregate consumption are cointegrated as the right hand side of equation (11) is stationary. The following proposition summarizes the result.

Proposition 7 *In the model of Bansal, Kiku and Yaron (2012), stock prices and aggregate consumption are cointegrated with cointegrating vector* $(1, -1)$ *and realized (log) stock price consumption ratio is a stationary process.*

Table 6: Stationarity of log price consumption ratio

I(1) test (Long-run risks)		*I(1) test (Habit)*	
PP (Z_t statistics)	-5.962	PP (Z_t statistics)	-26.752
1% critical value	-3.455	1% critical value	-3.430
DF-GLS	-3.737	DF-GLS	-3.560
1% critical value	-3.480	1% critical value	-2.580

We also simulate the long-run risks model for 948 periods (months) as in Bansal, Kiku and Yaron (2012) to confirm the stationarity. The left panel of Table 6 shows the unit root testing results by applying the PP test and the DF-GLS test to (log) price consumption ratio. Both test statistics are smaller than the corresponding 1% critical value, suggesting that realized stock price consumption ratios pass the unit root tests.

4.2 The habit model and production economy asset pricing models

Consider the habit model of Campbell and Cochrane (1999). Readers are referred to their paper on the details. Aggregate consumption contains a unit root. There is no analytical solution for the habit model. We simulate the habit formation model for 120, 000 months (as in Campbell and Cochrane (1999)) and then test the cointegration between realized quarterly (log) stock prices and (log) aggregate consumption. Using the PP and DF-GLS test, the right panel of Table 6 shows that realized price consumption ratios pass both tests as the null hypothesis is rejected.[18]

Here we interpret stocks as a claim to the consumption stream. This interpretation is present in Campbell and Cochrane (1999) as it is common in the equity premium literature; see Section D, p. 216 as well as their Table 2. The price of a claim to the consumption stream is cointegrated with aggregate consumption. One may prefer to interpret stocks as a claim to the dividend stream. In some asset pricing models, aggregate consumption is assumed not to be cointegrated with aggregate dividends. Thus, in these models, the price of a claim to the dividend stream is not necessarily cointegrated with aggregate consumption.

However, the assumption of no cointegration between consumption and dividends is not made based on empirical evidence but typically for the purpose of simplifying analysis. In the basic habit formation model of Campbell and Cocharane (1999), aggregate consumption and aggregate dividends are not cointegrated. They, nevertheless, say "*It would be better to make dividends and consumption cointegrated. We have explored a model in which the log dividend/consumption ratio is i.i.d. and the correlation of one-period dividend and consumption growth rates is low as in the data... A cointegrated model with a persistent log dividend/consumption ratio would be more realistic, but this modification would require an additional state variable*" (p. 217). Using the postwar US data, we indeed find that

[18] Figure 3 of Campbell and Cochrane (1999) shows that $\log(P_t) - \log(C_t)$ is approximately linear in the stationary state variable, i.e., consumption surplus ratio s_t. This also suggests the stationarity of $\log(P_t) - \log(C_t)$.

aggregate consumption and aggregate dividends are cointegrated. In stock pricing models that assume aggregate consumption is cointegrated with aggregate dividends (e.g., Adam, Marcet and Beutel (2017)), the price of a claim to the dividend stream will also be cointegrated with consumption. With this view, we tend to think that allowing no cointegration between consumption and dividends in models may not be a promising avenue to reconcile the new survey evidence.

In production-based asset pricing models with RE and full information, such as Jermann (1997), Boldrin, Christiano and Fisher (2001) and Croce (2014), the exogenous driving process is a productivity process which is assumed to contain a unit root. Agents are endowed with full information about the economy and can deduce the equilibrium mapping from the exogenous productivity process to endogenous variables. Both stock prices and consumption are endogenous variables. Different numerical methods may be employed to solve these models. Yet a common feature is that (log) stock price consumption ratio can be well approximated by a polynomial function of stationary state variables and is again stationary. Thus, realized stock prices and aggregate consumption are cointegrated with cointegrating vector $(1, -1)$ and agents have this knowledge as a consequence of having RE. The tests developed in Section 2 can be applied to these production-based models which appear inconsistent with the new survey finding.

5 Incomplete information models

Can models with incorporating non-rational (or extrapolative) consumption forecasts reconcile our survey evidence in Section 3.3? We firstly show consumption growth forecasts do not pass the conventional rationality test. This preliminary testing result lends some supports to asset pricing models with non-rational (or extrapolative) consumption forecasts. Yet we show that these models appear inconsistent with the evidence in Section 3.3, mainly due to agents' knowledge of the equilibrium pricing function.

5.1 RE tests of consumption growth forecasts

Using data covering 1981Q1 to 2017Q4, actual consumption growth rates (g_{ct}) is regressed on the 1-quarter ahead SPF median consumption growth forecast (g_{ct}^{e}) . We obtain

$$g_{ct} = \underset{(0.565)}{-0.423} + \underset{(0.214)}{1.340} g_{ct}^{e}, \qquad R^2 = 0.294, \tag{12}$$

where the numbers in the parenthesis are standard deviations. An F-test of the joint hypothesis that the intercept of (12) equals 0 and slope coefficient equals 1 yields a highly significant F-statistics of 8.36. The results reject that 1-quarter ahead SPF consumption growth forecasts are (full-information) rational forecasts. The same conclusion is reached by using a different forecasting horizon (i.e., 2-quarter ahead) or using mean price forecasts. However, as illustrated below, incorporating non-rational (or extrapolative) consumption growth forecasts alone cannot reconcile equity pricing models with the new evidence we documented. In what following we consider two types of incomplete information models in both of which agents possess the knowledge of equilibrium pricing function.

5.2 Sentiment-based models

Some papers introduce sentiment into asset pricing models, such as Jin and Sui (2018) and the exchange rate model of Yu (2013). As an example, suppose the representative agent's preferences are represented by the Epstein-Zin utility (6) and the actual exogenous driving processes are

$$\Delta c_{t+1} = \mu_c + \sigma_t \eta_{t+1}, \tag{13}$$

$$\sigma_{t+1}^2 = \overline{\sigma}^2 + \nu(\sigma_t^2 - \overline{\sigma}^2) + \sigma_w w_{t+1}, \tag{14}$$

$$\Delta d_{t+1} = \mu_d + \pi \sigma_t \eta_{t+1} + \varphi \sigma_t u_{d,t+1}. \tag{15}$$

Comparing (13) - (15) with (7) - (10) in the long-run risks model of Bansal, Kiku and Yaron (2012), we drop the persistent component x_t in the actual exogenous driving processes (because sentiment plays the role of the persistent component x_t).[19]

Now assume agents have misperception about the exogenous consumption and dividend

[19] Our proposition below is not affected by adding x_t in the exogenous driving processes, as is argued later.

process. They perceive consumption and dividend processes as

$$
\begin{aligned}
\Delta c_{t+1} &= \mu_c + a_t + \sigma_t \widehat{\eta}_{t+1}, && (16) \\
a_{t+1} &= \rho a_t + \varphi_e \sigma_t e_{t+1}, && (17) \\
\sigma^2_{t+1} &= \overline{\sigma}^2 + \nu(\sigma^2_t - \overline{\sigma}^2) + \sigma_w w_{t+1}, && (18) \\
\Delta d_{t+1} &= \mu_d + \phi a_t + \pi \sigma_t \eta_{t+1} + \varphi \sigma_t u_{d,t+1}, && (19)
\end{aligned}
$$

where $(\widehat{\eta}_{t+1}, e_{t+1})$ are i.i.d joint standard normal under agents' belief. a_t is an AR(1) process and does not appear in the true driving processes (called "sentiment"). When a_t is positive (negative), agents are optimistic (pessimistic). Assuming $0 < \rho < 1$. If $\rho = 1$, $\log(P_t/C_t)$ will have unbounded volatility (see equation (20) later), which is rejected by the data. Thus, we require $\rho < 1$. Note in this type of models, despite agents' misperception of the exogenous driving process, they know the equilibrium pricing function. The following result provides testable implications for the formation of stock price expectations.

Proposition 8 *Given agents' beliefs (16) - (19), agents' stock price forecasts* $E_i \log P_{i+j}$ *are cointegrated with their forecasts of aggregate consumption* $E_k \log C_{k+l}$ *with cointegrating vector* $(1, -1)$ *for arbitrary* i, j, k, $l > 0$.

Proof. Following Bansal, Kiku and Yaron (2012), the (approximate) analytical solution for price consumption ratio can be derived as

$$
\log(\frac{P_t}{C_t}) = A_0 + A_1 a_t + A_2 \sigma^2_t, \tag{20}
$$

where A_0, A_1, A_2 remain the same constant as in Bansal, Kiku and Yaron (2012). The trend growth rate of both stock prices forecasts and consumption forecasts are identical to μ_c (noting sentiment is a stationary process). Thus, stock price forecasts and consumption forecasts can be expressed as $E_i \log P_{i+j} = \log P_i + j\mu_c + s(i,j)$ and $E_k \log C_{k+l} = \log C_k + l\mu_c + \widetilde{s}(k,l)$ where $s(i,j)$ and $\widetilde{s}(k,l)$ are stationary terms and omitted. Let $E_i \log P_{i+j} - E_k \log C_{k+l}$ $= (\log P_t - \log C_k) + (j-l)\,\mu_c + s(i,j) - \widetilde{s}(k,l)$. It is stationary because $(\log P_t - \log C_k)$ is stationary.[20] ∎

[20] $(\log P_t - \log C_k) = (\log P_t - \log C_t) + (\log C_t - \log C_k)$ is stationary because $(\log P_t - \log C_t)$ is stationary (see equation (20)) and $(\log C_t - \log C_k)$ is stationary (since $\log C_t$ is I(1)).

Stock price forecasts depend on current stock prices and beliefs about the growth rate of stock prices. Similarly, consumption forecasts depend on current consumption and beliefs about the growth rate of consumption. Stock price forecasts and consumption forecasts are cointegrated because (1) realized stock prices and consumption are cointegrated and (2) beliefs about consumption growth and stock price growth rates are mean-reverting. If beliefs about consumption growth rates are not mean-reverting ($\rho = 1$), price consumption ratio will have unbounded volatility given agents' knowledge of the equilibrium pricing function.

If the consumption driving process (13) - (15) contains a persistent and stationary component x_t as in Bansal, Kiku and Yaron (2012), (log) price consumption ratio will be a linear function of x_t, a_t and σ_t^2 with constant coefficients. The above proposition will still hold because realized price consumption ratios are stationary and the trend growth rate of both stock prices forecasts and consumption forecasts are identical to μ_c.

5.3 Learning about consumption dynamics

Many asset pricing models maintain the assumption of RE but assume agents have incomplete information and learn about the exogenous consumption process.[21] In this type of learning models, agents know the equilibrium pricing mapping and form RE about stock prices. Suppose the representative agent's preferences is represented by the Epstein-Zin utility (6). For illustration, the consumption process is

$$\Delta c_{t+1} = \mu_c + \overline{\sigma}\eta_{t+1}, \tag{21}$$

where η_{t+1} is an i.i.d process. Agents do not know the consumption growth rate but know the constant variance $\overline{\sigma}^2$. Agents learn μ_c over time and beliefs about μ_c is updated by

$$\mu_{c,t} = \mu_{c,t-1} + g_t\left(\Delta c_t - \mu_{c,t-1}\right). \tag{22}$$

Assuming constant gain or Kalman filter learning (under steady state variance ratio) is used, i.e., $g_t = g \in (0,1)$. Substituting (21) into (22) yields $\mu_{c,t} = (1-g)\mu_{c,t-1} + g\left(\mu_c + \overline{\sigma}\eta_t\right)$ which is a stationary process.

[21] Collin-Dufresne, Johannes and Lochstoer (2016) and Johannes, Lochstoer and Mou (2016) are examples of asset pricing models along this line.

Proposition 9 *Given the beliefs (21), agents' forecasts of stock prices $E_i \log P_{i+j}$ are cointegrated with their forecasts of aggregate consumption $E_k \log C_{k+l}$ with cointegrating vector $(1, -1)$ for arbitrary i, j, k, $l > 0$.*

Proof. The RE version of the model here is a special case of Bansal, Kiku and Yaron (2012) with setting $\rho = 0$, $\nu = 0$, $x_t = 0$, $\sigma_t = \overline{\sigma}$, $\sigma_w = 0, \varphi_e = 0$. From the RE version to the learning model, we replace the actual growth rate of consumption μ_c by agents' beliefs about consumption growth rate $\mu_{c,t}$ in the analytical solution. The (analytical) solution for log price consumption ratio in the learning model is

$$\log(\frac{P_t}{C_t}) = \widetilde{A}_{0,t} + \widetilde{A}_2 \overline{\sigma}^2, \tag{23}$$

where $\widetilde{A}_{0,t} = \frac{1}{1-\kappa_{1,t}} \left(\log \delta + \kappa_{0,t} + \left(1 - \frac{1}{\psi}\right) \mu_{c,t} + \kappa_{1,t} \widetilde{A}_2 \overline{\sigma}^2\right)$, $\widetilde{A}_2 = -\frac{(\gamma-1)\left(1-\frac{1}{\psi}\right)}{2}$, $\kappa_{0,t} = \log(1 + \exp(\overline{z}_t)) - \kappa_{1,t}\overline{z}_t$, $\kappa_{1,t} = \frac{\exp(\overline{z}_t)}{1+\exp(\overline{z}_t)}$, $\overline{z}_t = \widetilde{A}_{0,t}\left(\overline{z}_t\right) + \widetilde{A}_2 \overline{\sigma}^2$. Note $\widetilde{A}_{0,t}$ is a nonlinear function of $\mu_{c,t}$. Using Taylor expansion, the right hand side of the process (23) can be well approximated by a polynomial function of the AR(1) process $\mu_{c,t}$ and is again a stationary process.[22] Again, $(E_i \log P_{i+j} - E_k \log C_{k+l})$ is stationary because realized stock price and consumption are cointegrated and the trend growth rate of stock price forecasts and consumption forecasts are identical to each other. ■

Notice Proposition 8 and 9 hold even if stock price forecasts and consumption forecasts are made at different dates and/or over different forecasting horizons. While survey consumption growth forecasts appear to be non-rational, incorporating this feature alone into asset pricing models cannot break the cointegration relationship between stock price forecasts and consumption forecasts. This aspect of the formation of stock price expectations in those models appears inconsistent with the evidence in Section 3.3.

6 Asset pricing models with heterogeneous beliefs

Can deviation from the representative agent assumption reconcile the new survey evidence in Section 3.3? This section shows models with heterogeneous beliefs and a willingness to

[22] If agents learn use least squares (i.e., $g_t = 1/t$), $\mu_{c,t}$ will converge to μ_c and $\widetilde{A}_{0,t}$ will converge to a constant.

"agree to disagree" do not necessarily reproduce this evidence. In these models, all agents can deduce and agree on the equilibrium pricing function and state contingent stock prices. This knowledge usually requires strong informational assumptions. For instance, all agents' beliefs about fundamentals and preferences etc are common knowledge.

Consider the model of Ehling,Graniero and Heyerdahl-Larsen (2018) (EGH henceforth) which is a continous-time overlapping generations economy. They study asset prices and portfolio choice by incorporating agents' learning from own experience about output process in a dynamic complete market setting. There are different cohorts who are born at different times and have heterogeneous beliefs about fundamentals, i.e., exogenous aggregate output process Y_t. The true process for Y_t is $dY_t/Y_t = \mu_Y dt + \sigma_Y dz_t$ where z_t is a standard Brownian motion. Agents disagree on this process and perceive that

$$dY_t/Y_t = \hat{\mu}_{s,t} dt + \sigma_Y dz_{s,t},$$

where the subscript s represents the cohort born at time s, $\hat{\mu}_{s,t}$ agents' perceived output growth rate, $z_{s,t}$ denotes a Brownian motion under the belief of an agent born at time s. Agents know the standard deviation of output σ_Y.

Denote by $E_i^s \log P_{i+j}$ stock price forecasts made by cohort s at time i and over horizon j; similarly for the forecast of aggregate consumption $E_k^s \log C_{k+l}$.[23] Define $\overline{E}_i \log P_{i+j}$ the average of stock price forecasts across all agents made at time i and over horizon j; similarly for average forecasts of aggregate consumption $\overline{E}_k \log C_{k+l}$.

Proposition 10 *In the EGH model, the average stock price forecasts across all agents $\overline{E}_i \log P_{i+j}$ is cointegrated with the average consumption forecasts $\overline{E}_k \log C_{k+l}$ with cointegrating vector $(1, -1)$ for arbitrary i, j, k, $l > 0$.*

Proof. All agents know (1) aggregate consumption C_t equals to Y_t ($C_t = Y_t$) each period and that consumption process is identical to output process and (2) the mapping from output to equilibrium stock prices, i.e., $P_t = \frac{1-\omega}{\rho+\nu(1-\beta)} Y_t$ (equation (B60) in EGH), and hence $P_t = \frac{1-\omega}{\rho+\nu(1-\beta)} C_t$. Thus, the perceived consumption process for cohort s is $dC_t/C_t = \hat{\mu}_{s,t} dt + \sigma_Y dz_{s,t}$. Agents' heterogeneous beliefs about fundamentals Y_t (or C_t) are carried over to stock prices P_t, which reads as $dP_t/P_t = \hat{\mu}_{s,t} dt + \sigma_Y dz_{s,t}$. Thus, every cohort perceives that

[23] In EGH, S_t stands for stock prices. Here we use a different notation P_t.

the growth rate of C_t and S_t are the same stochastic process despite heterogeneous beliefs about the growth rate across cohorts.

For cohort s, stock price forecasts $E_i^s \log P_{i+j}$ depends on realized stock prices P_i and perceived stock price growth rates $\hat{\mu}_{s,i}$. Similarly, consumption forecasts $E_k^s \log C_{k+l}$ depend on realized consumption C_k and perceived consumption growth rates $\hat{\mu}_{s,k}$. First, note $P_i = \frac{1-\omega}{\rho+\nu(1-\beta)} Y_i$ and $C_k = Y_k$. Clearly, $logP_i$ is cointegrated with $logC_k$ with cointegrating vector $(1,-1)$ because $logY_i$ and $logY_k$ is cointegrated with cointegrating vector $(1,-1)$. Second, Proposition 1 of EGH implies that as $t \to \infty$, $\hat{\mu}_{s,t}$ and $\hat{\mu}_{s,k}$ will converge almost surely to the true output growth rate μ_Y. So $E_i^s \log P_{i+j}$ is cointegrated with $E_k^s \log C_{k+l}$ with cointegrating vector $(1,-1)$ for every cohort s. Taking the two features together, the average stock price forecasts $\overline{E}_i \log P_{i+j}$ is cointegrated with $\overline{E}_k \log C_{k+l}$ with cointegrating vector $(1,-1)$. $\blacksquare$

Although agents have heterogeneous beliefs about consumption (or output), the knowledge of the equilibrium pricing function implies that stock price forecasts made by every individual is cointegrated with its aggregate consumption forecasts. Thus, the average stock price forecasts across agents is cointegrated with the average consumption forecasts. This aspect of the formation of stock price forecasts in this model appears inconsistent with our survey evidence in Section 3.3 or Appendix B.

7 Adaptive learning models

The survey evidence casts some doubt on stock pricing models which assume agents possess the knowledge of equilibrium pricing function, like those studied in Section 4, 5 and 6. This knowledge usually requires strong informational assumptions on agents, such as other agents' beliefs and preferences etc; see e.g., Adam and Marcet (2011). The long-run component of stock price forecasts is anchored by consumption forecasts in these models.

Unlike these models, agents are assumed not having this knowledge in asset pricing models with adaptive learning, e.g., Adam, Marcet and Beutel (2017, henceforth AMB). Unit root econometrics can also be applied to test the expectation formation in this type of models. Deriving the testable implications requires using agents' perceived and actual law of motion for model variables. We show the typical specifications of stock price beliefs in adaptive learning models (including AMB) imply that stock price forecasts are cointegrated with consumption forecasts. Thus, relaxing this knowledge alone is insufficient for models to reproduce the survey evidence in Section 3.3. Yet Section 8 shows this evidence can be reconciled with a modification of the belief specification in AMB together with assuming agents' lack of the knowledge of the equilibrium pricing function.

7.1 Setup of the AMB Model

We firstly consider the endowment economy asset pricing model of Adam, Marcet and Beutel (2017, henceforth AMB). Readers are referred to their paper for details. It quantitatively replicates many asset-pricing moments and endogenously generates boom-bust asset price dynamics. Importantly, the model is consistent with survey expectation data that price dividend ratio comoves positively with survey return expectations which cannot be matched by RE models (shown in AMB).

It is an infinite horizon endowment economy model where agents receive dividends and exogenous wage income. They decide on stock holding and consumption for each period. Dividends grow at a constant rate

$$\log D_t = \log \beta^D + \log D_{t-1} + \log \varepsilon_t^D, \tag{24}$$

where $\beta^D \geq 1$ stands for the mean growth rate and $\ln \varepsilon_t^D$ an *i.i.d.* growth innovation described further below. The exogenous wage income process W_t is

$$\log\left(1 + \frac{W_t}{D_t}\right) = (1-p)\log(1+\rho) + p\log\left(1 + \frac{W_{t-1}}{D_{t-1}}\right) + \ln \varepsilon_t^W, \tag{25}$$

where $1+\rho$ is the average consumption-dividend ratio and $p \in [0,1)$ its quarterly persistence. The innovations are given by

$$\begin{pmatrix} \log \varepsilon_t^D \\ \log \varepsilon_t^W \end{pmatrix} \sim iiN\left(-\frac{1}{2}\begin{pmatrix} \sigma_D^2 \\ \sigma_W^2 \end{pmatrix}, \begin{pmatrix} \sigma_D^2 & \sigma_{DW} \\ \sigma_{DW} & \sigma_W^2 \end{pmatrix}\right), \tag{26}$$

with $E\varepsilon_t^D = E\varepsilon_t^W = 1$. Aggregate consumption is

$$C_t = W_t + D_t. \tag{27}$$

Agents' beliefs and preferences etc are not common knowledge. They cannot correctly deduce the equilibrium law of motion for asset prices. Instead, they form subjective price belief and learn from market prices.

7.2 Testable implications

Consider the baseline specification for subjective stock price beliefs in AMB

$$
\begin{aligned}
\Delta \log P_{t+1} &= \log \beta_{t+1} + \log \epsilon_{t+1}, &(28)\\
\log \beta_{t+1} &= \log \beta_t + \log \nu_{t+1}. &(29)
\end{aligned}
$$

Agents are uncertain and learn about $\log \beta_t$ over time. They are assumed to know the exogenous driving processes, (24), (25) and (27). These equations give that

$$
\log C_t = \log(D_t + W_t) = \log D_t + \log(1 + \frac{W_t}{D_t}). \tag{30}
$$

On the one hand, agents' perceived law of motion (PLM) for stock prices (28) - (29) is I(2). On the other hand, the consumption process (30) is I(1). One may think that stock price forecasts will not be cointegrated with consumption forecasts. Yet the following proposition shows this thinking turns out to be incorrect.

Proposition 11 *Suppose agents' perceived law of motion for stock prices is (28) - (29), agents' stock price forecasts $E_i \log P_{i+j}$ are cointegrated with consumption forecasts $E_k \log C_{k+l}$ with cointegrating vector $(1, -1)$ for arbitrary i, j, k, $l > 0$.*

Proof. Denote by $\log m_i$ agents' belief about the growth rate of stock prices at period i. Given agents' PLM (28) - (29), we have $E_i \log P_{i+j} = \log P_i + j \log m_i$.[24] Consumption forecasts are $E_k \log C_{k+l} = E_k(\log D_{k+l} + \log(1 + \frac{W_{k+l}}{D_{k+l}})) = \log D_k + l \log \beta^D + E_k(\log(1 + \frac{W_{k+l}}{D_{k+l}}))$. So $E_i \log P_{i+j} - E_k \log C_{k+l} = (\log P_i + j \log m_i) - (\log D_k + l \log \beta^D + E_k(\log(1 + \frac{W_{k+l}}{D_{k+l}}))) = (\log P_i - \log D_k) + (j \log m_i - l \log \beta^D) - E_k(\log(1 + \frac{W_{k+l}}{D_{k+l}}))$. $(E_i \log P_{i+j} - E_k \log C_{k+l})$ is stationary for three reasons. First, Lemma 1 of AMB shows that without uncertainty $\log m_t$ is mean-reverting and converges to the corresponding RE value $\log \beta^D$. In particular, Appendix

[24] Note stock price forecasts in the model contain a unit root because realized stock prices contain a unit root (sharing a common trend with dividend) and $\log m_t$ is stationary.

$A10$ of AMB shows that under certain condition, the ALM for $\log m_t$ is a second-order difference equation and the eigenvalues determining the stability of the solution are inside the unit circle. Thus, $\log m_t$ is a stationary process. Second, price dividend ratio is stationary; see Proposition 3 of AMB.[25] Note $\log P_i - \log D_k = (\log P_i - \log P_k) + (\log P_k - \log D_k)$. $(\log P_i - \log D_k)$ is stationary because both $(\log P_i - \log P_k)$ and $(\log P_k - \log D_k)$ are stationary. Third, $E_k\left(\log(1 + \frac{W_{k+l}}{D_{k+l}})\right)$ is stationary because $\log(1 + \frac{W_{k+l}}{D_{k+l}})$ is stationary. ■

Deriving the tests of expectation formation requires utilizing both agents' PLM and the actual law of motion for stock prices. Agents' PLM for stock prices gives us the expression for stock price forecasts as a function of current stock prices and agents' belief about the growth rate of stock prices. Stock price forecasts and consumption forecasts are cointegrated because realized stock prices and consumption are cointegrated and agents' belief about the long-run growth rate of stock prices will mean-revert to the long-run growth rate of consumption, despite agents' lack of knowledge of both features.[26]

Many adaptive learning models assume agents learn about detrended stock prices e.g., Carceles-Poveda and Giannitsarou (2008). This implies agents know exactly the evolution of the trend growth rate of stock prices and consumption as in full-information RE models.[27] Thus, in these models, stock price forecasts are cointegrated with consumption forecasts too. To sum up, the typical specifications on subjective stock price beliefs in adaptive learning models appear incompatible with the evidence presented in Section 3.3. Yet Section 5 shows a modification of the belief specification in AMB reconciles the new survey evidence.

8 Reconciling models with the new survey evidence

How can we break the tight link between the long-run component of stock price forecasts and consumption forecasts in asset pricing models? This section discusses potential resolutions for reconciling models with the new survey evidence.

[25] In AMB, the stationarity of the price dividend ratio is essential for rejecting RE using survey expectation data and for calculating the statistical moments of price dividend ratio in the learning model.

[26] AMB considers another general belief specification which features mean-reversion in stock price growth rates. In this case, stock price forecasts remain cointegrated with consumption forecasts; an earlier version of the paper contains the proof which is also available upon request.

[27] This can be shown following Section 7 of Kuang and Mitra (2016).

8.1 Adding sentiment directly to stock price forecasts

Incorporating non-rational (or extrapolative) consumption expectations or allowing heterogeneous beliefs with a willingness to "agree to disagree" among agents does not break the cointegration between stock price forecasts and consumption forecasts, as is shown in Section 5 and 6. The long-run component of stock price forecasts is anchored by consumption forecasts in these models via agents' knowledge of the equilibrium pricing function. Thus, our survey evidence casts some doubt on the modeling of expectation formation in these models. The discrepancy between stationary realized price consumption ratios and the corresponding nonstationary forecasts creates a challenge on the asset pricing models which assume agents possess this strong knowledge.

One way to reconcile the evidence appears to require the representative agent not having the knowledge of the equilibrium pricing function (as in e.g., AMB) or a sufficiently large fraction of investors not having this knowledge in models with heterogeneous beliefs. Due to agents' lack of this knowledge, it is possible for an asset pricing model to simultaneously produce stationary realizations and non-stationary forecasts of price to consumption ratios.

Relaxing this knowledge alone is insufficient to reproduce the survey evidence because Section 7 shows the typical specifications of stock price beliefs in adaptive learning models imply that stock price forecasts are cointegrated with consumption forecasts. Our empirical evidence thus suggests that major asset pricing models may have missed an important persistent component which independently drives stock price forecasts but not consumption forecasts. We show below that in a representative agent setting, the survey evidence is reproduced when we modify the belief specification in AMB by adding a non-stationary sentiment or judgment component (which is not cointegrated with consumption forecasts) *directly* to stock price forecasts together with assuming the agent's lack of knowledge of the equilibrium pricing function (as in AMB). Later we also provide some discussions on reconciling the survey evidence in models with heterogeneous beliefs.

We modify the representatitve agent's subjective stock price forecasts in the AMB model (discussed in Section 7) in the following way. The agent's subjective stock forecasts ($\log P^e_{t+1}$) consist of two components. The first component of stock price forecasts (denoted by $E_t \log P_{t+1}$) is produced as in AMB. Specifically, the agent has an econometric model of stock prices but does not know the parameters of the model. She needs to learn about the parameters using historical stock price data and then makes forecasts using the estimated model. The second component is new and denoted by $\log \gamma_t$. Denote by $\log P^e_{t+1}$ the agent's forecast of stock prices in period $t+1$ made at period t. Mathematically, the agent's stock price forecasts are

$$\log P^e_{t+1} = E_t \log P_{t+1} + \log \gamma_t, \tag{31}$$
$$\log \gamma_t = \log \gamma_{t-1} + \log \xi_t. \tag{32}$$

$\log \xi_t$ is assumed as an i.i.d process. When $\log \gamma_t$ is positive (or negative), the agent is more optimistic (or pessimistic) than the stock price forecasts produced by the econometric model.

Reconciling our survey evidence requires two crucial assumptions on $\log \gamma_t$. **Assumption 1**: $\log \gamma_t$ is an I(1) process. **Assumption 2**: $\log \gamma_t$ is independent of or at least not cointegrated with the agent's aggregate consumption forecasts. If either of the two assumptions is violated, that is, if $\log \gamma_t$ is stationary or cointegrated with consumption forecasts, stock price forecasts will be cointegrated with consumption forecasts, which is inconsistent with our survey evidence.

Two remarks are as follows. First, $\log \gamma_t$ in equation (31) is assumed to be a random walk process. It can be specified as a more general $I(1)$ process and the proposition below will not be affected. Second, for the purpose of replicating the evidence, assuming $\log \gamma_t$ is private information or common knowledge does not matter.

An interpretation of the sentiment variable $\log \gamma_t$ is (the "guesswork" component of) judgment made by forecasters. This is known as "add-factoring" the forecast in the forecasting community. For instance, Bullard, Evans and Honkapohja (2008) examines the role of agents' judgmental adjustment to forecasts in learning models.[28] Further research on the formation of stock price expectations would help to understand better the nature of this variable which drives stock price forecasts but not consumption forecasts.

The first component of stock price forecasts ($E_t \log P_{t+i}$) is generated from the following forecasting model

$$\Delta \log P_t = \log \beta_t + \log \epsilon_t, \tag{33}$$
$$\log \beta_t = \left(1 - \eta_\beta\right) \log \beta^D + \eta_\beta \log \beta_{t-1} + \log \nu_t, \tag{34}$$

[28] They show this may lead to self-fulfilling fluctuations in New Keynesian models. Alternatively, $\log \gamma_t$ may be called "expectation shocks", see Milani (2011) for an estimated New Keynesian model with learning and expectation shocks.

where $\log \epsilon_t$ and $\log \nu_t$ are i.i.d. innovations. $\log \xi_t$, $\log \epsilon_t$, and $\log \nu_t$ are independent to each other. The agent's belief about $\log \beta_t$ is updated by

$$\begin{aligned}\log m_t &= \left(1-\eta_\beta\right)\log \beta^D + \eta_\beta \log m_{t-1} \\ &\quad + g\left(\log P_{t-1} - \log P_{t-2} - \log m_{t-1}\right), \end{aligned} \tag{35}$$

where g is the Kalman gain parameter.

Proposition 12 *Given the agent's price belief (31) - (34), we have (1) the agent's stock price forecasts* $\log P^e_{i+j}$ *are I(1), consistent with the evidence in Section 3.2; (2) stock prices forecasts* $\log P^e_{i+j}$ *and consumption forecasts* $\log C^e_{k+l}$ *are not cointegrated for arbitrary* i, j, k, $l > 0$, *consistent with the survey evidence in Section 3.3; (3) forecasts of stock prices, i.e.,* $\log P^e_{i+j}$ *and* $\log P^e_{k+l}$, *are cointegrated for (a)* $i = k$, $j \neq l$ *or (b)* $i \neq k$, $j > 0$, $l > 0$, *consistent with the evidence in Section 3.4.*

Proof. (1) Given (31) - (34), stock price forecasts are $\log P^e_{i+j} = E_i \log P_{i+j} + \log \gamma_i = \log P_i + j \log \beta^D + \widetilde{s}(i,j) + \log \gamma_i$, where $\widetilde{s}(i,j)$ is a stationary term and omitted because it is irrelevant for the proof. Denote by L the lag operator. Taking the difference of $\log P^e_{i+j}$ yields $(1-L)\log P^e_{i+j} = (E_i \log P_{i+j} + \log \gamma_i) - \left(E_{i-1}\log P_{i-1+j} + \log \gamma_{i-1}\right) = (\log P_i + j \log \beta^D + \widetilde{s}(i,j) + \log \gamma_i) - \left(\log P_{i-1} + j\log \beta^D + \widetilde{s}(i-1,j) + \log \gamma_{i-1}\right) = \Delta \log P_i + \log \xi_i + \Delta \widetilde{s}(i,j)$. Given $\Delta \log P_i$ is stationary (as in the data) and $\Delta \widetilde{s}(i,j)$ is stationary, we have shown stock price forecasts are I(1), consistent with the evidence in Section 3.2.

(2) The consumption forecasts is again the same as that in the proof of Proposition 11. Let $\log P^e_{i+j} - \log C^e_{k+l} = (\log P_i + j\log \beta^D + \widetilde{s}(i,j) + \log \gamma_i) - (\log D_k + l \log \beta^D + E_k(\log(1+\frac{W_{k+l}}{D_{k+l}})))$ $= (\log P_i - \log D_k) + \log \gamma_i + (j-l)\log \beta^D + \widetilde{s}(i,j) - E_k(\log(1+\frac{W_{k+l}}{D_{k+l}}))$. Because $\log \gamma_i$ is I(1) (Assumption 1) and not cointegrated with consumption forecasts (Assumption 2), we have shown that $\left(\log P^e_{i+j} - \log C^e_{k+l}\right)$ is I(1), consistent with the evidence in Section 3.3.

(3) Let $\log P^e_{i+j} - \log P^e_{k+l} = (E_i \log P_{i+j} + \log \gamma_i) - (E_k \log P_{k+l} + \log \gamma_k) = (\log P_i + j \log \beta^D + \widetilde{s}(i,j)) - (\log P_k + l \log \beta^D + \widetilde{s}(k,l))$. It is stationary because $(\log P_i - \log P_k)$, $\widetilde{s}(i,j)$ and $\widetilde{s}(i,k)$ are stationary. ■

In a representative agent setting, introducing a non-stationary sentiment $\log \gamma_t$ – which is not cointegrated with consumption forecasts – directly to stock price forecasts is the key to break the tight link between the trend component of stock price forecasts and consumption forecasts and reconcile the new survey evidence.

Can the model still produce a stationary realized price to consumption (or dividend) ratio when we add a non-stationary sentiment variable into stock price forecasts? Due to agents' lack of knowledge of the equilibrium pricing function, this type of models can simultaneously produce non-stationary forecasts of stock price to consumption (or dividend) ratios as well as stationary realized stock price to consumption (or dividend) ratios. In the AMB model, when expectations about stock price appreciation are sufficiently high, the wealth effect becomes as strong as (or even stronger than) the substitution effect. Investors' stock demand and stock prices will start to decline which leads to mean-reversion of the stock price to consumption (or dividend) ratio. A work in progress incorporates this type of sentiment shock into the model of Adam, Marcet and Nicolini (2016) and evaluates quantitatively its role for understanding stock price dynamics by estimating the stock pricing model with learning and stock price expectations data.

8.1.1 Constructing and testing the sentiment shock

We illustrate how to empirically construct a time series of this new component of stock price forecasts $\log \gamma_t$ (in a representative agent setting). We show the sentiment shock series does not pass standard unit root tests and is not cointegrated with consumption forecasts, confirming the two assumptions we made earlier. This exercise isolates the component of stock price forecasts which contains a unit root and is not cointegrated with consumption forecasts.

Assuming $\eta_\beta = 0$ which corresponds to the benchmark specification of AMB. Equation (35) gives a simple belief updating equation for stock price growth rates

$$\log m_t = \log m_{t-1} + g\left(\log P_{t-1} - \log P_{t-2} - \log m_{t-1}\right), \tag{36}$$

where g is the gain parameter.[29] This gives, for instance, the first component of 1-year ahead stock forecasts as $\log P_t + 2\log m_t$; note we consider a semi-annual frequency. On top of this, the agent adds a nonstationary new component (i.e., judgment or sentiment) $\log \gamma_t$ which follows a random walk process. Thus, the agent's 1-year ahead stock price forecast is $\log P_t + 2\log m_t +$ $\log \gamma_t$, which will be proxied by survey expectations data.

[29] The isolated sentiment shock series may be different if investors use a different econometric model to produce the first component of stock price forecasts. We do not explore this here as our exercise aims to provide an illustration (using the AMB specification which has good quantitative performance).

The agent's initial belief about semi-annual stock price growth rates is set to 4.5% (which is the average value in the historical stock price data taken from Adam, Marcet and Nicolini (2016)). We then substitute actual stock prices data over the period 1981Q2 - 2017Q2 into the belief updating rule (36). The gain value is set to 0.0262 taken from AMB.[30] With these, we obtain a time series of the first component of stock price forecasts. The sentiment shocks can be constructed as the difference between 1-year ahead Livingston median stock price forecasts and the first component of price forecasts (up to a stationary or *i.i.d* measurement error).[31]

Two properties of the sentiment shock are established. First, Table 7 reports the statistics of PP and DF-GLS testing of the constructed sentiment shock along with 10% critical values. Both tests suggest the null hypothesis that this series contains a unit root cannot be rejected at 10% level. Second, using Johansen test, we find no cointegration between the sentiment shock and consumption forecasts (the results are omitted here and available upon request).

Table 7: Unit root tests of the sentiment shock

PP Z_t stat.	10% critical value	DF-GLS	10% critical value
-1.5577	-1.6129	-1.701	-1.903

Note sentiment here is incorporated into price forecast, while in the literature it is usually added to fundamentals (e.g., consumption or dividend). It is the difference between survey expectation forecasts and the forecasts generated by an econometric model made by agents. Thus, this sentiment shock may be substantially different from e.g., the Consumer Sentiment Index in the Michigan Survey of Consumers or the Consumer Confidence Index provided by the Conference Board.

8.2 Further discussions

Several issues may deserve some further discussions. First, for models with heterogeneous beliefs, Section 6 shows if all agents have the knowledge of the equilibrium pricing function

[30]The choice of the gain parameter can be alternatively determined by estimating stock pricing models with utilizing stock price expectations data. The constructed sentiment shock may be different if investors use a different gain parameter. But we find the testing results are robust to alternative choices of the gain parameter.

[31]Note the integration and cointegration property of the constructed sentiment shock is not affected by measurement errors as long as the measurement errors are stationary or *i.i.d.* over time.

and "agree to disagree" about the fundamentals, the average forecast of stock prices across agents is still cointegrated with the average forecast of consumption. Thus, reconciling the survey evidence in a heterogeneous agents setting would require no cointegration between stock price forecasts and consumption forecasts for a sufficiently large fraction of investors. This may in turn require (1) assuming these investors do not have the knowledge of the equilibrium pricing function and (2) for each of these investors, a non-stationary sentiment (or judgment) variable independently drives her stock price forecasts but not consumption forecasts. With the availability of surveys containing individual stock price forecasts and consumption forecasts, applying our tests at the level of individual investors can help to understand better the degree and nature of belief heterogeneity among investors, such as the relationship between the forecasts made by investors with different characteristics (e.g., education, income) and if a common (or different) non-stationary sentiment variable drives stock price forecasts made by different investors.

Second, can the survey evidence be reconciled by adding a non-stationary sentiment component to consumption forecasts (but not stock price forecasts) in the AMB model and assuming investors do not have the knowledge of the equilibrium pricing function? Consider firstly the models in Section 5.2 which assume investors have the knowledge of the equilibrium pricing function but misperception about the consumption process. Adding a non-stationary sentiment component to consumption (like setting $\rho = 1$ in equations 16 - 17) will yield price consumption ratios with unbounded volatility; see equation (20). The AMB model with this alternative belief specification – together with assuming lack of knowledge of the equilibrium pricing function – will amplify the fluctuations of price consumption ratios as the agent's price growth beliefs will fluctuate around the corresponding value under the case with exact knowledge. In this case, the price consumption ratios will have unbounded volatility too. Thus, this avenue appears not promising to reconcile the evidence.

Third, some models of stock prices, e.g., De Long, Shleifer, Summers and Waldmann (1990) and Scheinkman and Xiong (2003), avoid modelling consumption so that consumption is not a component for understanding stock price movement. Similarly, consumption is not an important driving force of stock prices in the X-CAPM model of Barberis, Greenwood, Jin and Shleifer (2015). We note these models are not inconsistent with the new survey evidence here. However, these models are developed mainly for theoretical purposes and the feature that aggregate consumption does not influence (or influence little) on aggregate stock prices may be viewed by some economists as a little extreme.

Fourth, one may think structural breaks is a potential reason why stock price forecasts and consumption forecasts are not cointegrated. The focus of our paper is testing expectation formation in major asset pricing models which do not usually include structural breaks. Yet we think our testing results here are robust to considering some types of structural breaks. For instance, the 2007 - 2008 Global Financial Crisis may be viewed as a structural break that the trend growth rate of consumption is reduced. Rational agents will observe this decrease and the trend growth rate of equilibrium stock prices will have a one-to-one decline. The cointegration relation between stock prices and consumption (including the cointegrating vector) is unaltered. Thus, our evidence on the no cointegration between stock price forecasts and consumption forecasts may not be reconciled by allowing this type of structural break and assuming agents possess the knowledge of the equilibrium pricing function. Similarly, this conclusion would still hold if the Financial Crisis is viewed as, for example, a shift in the level of the trend of consumption.

9 Conclusion

The paper demonstrates the usefulness of unit root econometrics in analyzing expectations data and testing expectation formation in financial and macroeconomic models with various informational assumptions. Models usually impose a large amount of cointegration restrictions, e.g., between forecasts of endogenous and exogenous variables or among forecasts of different endogenous variables. The tests developed in the paper utilize these restrictions.

We show median (or mean) survey stock price forecasts are not cointegrated with median (or mean) consumption forecasts. This evidence is robust to different sources of expectations data, forecasting horizons, statistical tests, using median or mean forecasts for testing, and using stock price forecasts data which is made at different dates from consumption forecasts.

The evidence casts some doubt on the modeling of expectation formation in a wide range of asset pricing models which assume agents possess the knowledge of equilibrium pricing function, including full-information RE models, incomplete information models with non-rational (or extrapolative) consumption forecasts, and models with heterogeneous beliefs and a willingness to "agree to disagree". Obtaining this knowledge by agents within the models usually requires strong informational assumptions on them, e.g., all agents' beliefs and preferences etc need to be common knowledge (see e.g., Adam and Marcet, 2011). Due to this exact knowledge, the long-run component of stock price forecasts is anchored by consumption forecasts in these models.

Relaxing agents' knowledge of the equilibrium pricing function (as in e.g., Adam, Marcet and Beutel, 2017) appears necessary to reconcile the new evidence. The evidence suggests major asset pricing models may miss an important persistent component which independently drives stock price forecasts but not consumption forecasts. In a representative agent setting, one way to reproduce the new evidence is incorporating a non-stationary sentiment (or judgment) component directly to subjective stock price forecasts together with assuming the agent's lack of knowledge of the equilibrium pricing function. Or in models with heterogeneous beliefs, the key ingredients for reconciling our survey evidence may be (1) a sufficiently large fraction of investors who do not have the knowledge of the equilibrium pricing function and (2) a non-stationary sentiment (or judgment) variable which independently drives subjective stock price forecasts of these investors but not consumption forecasts.

The formation of stock price expectations can be further explored in two dimensions. First, aggregate dividend forecasts and stock price forecasts are cointegrated in various asset pricing models considered in the paper. These can be tested with the availability of high quality data on aggregate dividend forecasts with a sufficiently long sample.[32] Second, the degree and nature of belief heterogeneity among investors can be studied further by applying the tests at the level of individual investors with the availability of surveys containing individual stock price forecasts and aggregate consumption forecasts (or dividend forecasts).

Applying unit root econometrics to expectations data – as is done in the paper – may be a promising avenue to provide useful guidance on the modeling of expectation formation. The tests developed in the paper can be applied in or adapted to other settings. First, while the tests are applied using mean or median survey forecasts, they can be applied using individual level data (where available) or experimental data too. The tests can be employed to study the relationship between forecasts made by economic agents with different characteristics (e.g., age, education, income, occupation) and to guide the modeling of expectation formation in models with heterogeneous beliefs. Second, in the presence of structural breaks, financial and macroeconomic models impose cointegration restrictions among forecasts of model variables. Our tests can be adapted to test and guide the modeling of expectation formation in this

[32] We do not pursue this analysis for the reason discussed in the Introduction.

setting.

Third, they can be implemented in other models, such as macroeconomic models and exchange rate models. An ongoing project finds that using median or mean SPF forecast data, output forecasts are not cointegrated with consumption forecasts (or investment forecasts). This evidence appears incompatible with a wide range of stochastic growth models, such as full-information RE models (including news shocks models) and incomplete-information models. The evidence may suggest households do not have the knowledge of the balanced growth path when making macroeconomic forecasts. Moreover, there appear independent driving forces for forecasts of different macroeconomic variables which are missed in typical business cycle models. We leave these for future work.

References

Adam, Klaus and Albert Marcet (2011). "Internal Rationality, Imperfect Market Knowledge and Asset Prices." Journal of Economic Theory 146, 1224 - 1252.

Adam, Klaus, Albert Marcet, and Johannes Beutel (2017): "Stock Price Booms and Expected Capital Gains", American Economic Review, vol 107(8), 2352-2408.

Adam, Klaus, Albert Marcet, and Juan Pablo Nicolini (2016). "Stock Market Volatility and Learning", Journal of Finance (1), 33 - 82.

Adam, Klaus, Dmitry Matveev, and Stefan Nagel (2018): "Do Survey Expectations of Stock Returns Reflect Risk-Adjustments?", NBER Working Paper.

Bansal, Ravi and Amir Yaron (2004): "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles", Journal of Finance, vol 59(4), 1481-1509.

Bansal, Ravi, Dana Kiku and Amir Yaron (2012): "An Empirical Evaluation of the Long-Run Risk Model for Asset Prices", Critical Finance Review, vol. 1, 183-221.

Barberis, Greenwood, Jin and Shleifer (2015): "X-CAPM: An Extrapolative Capital Asset Pricing Model", Journal of Financial Economics, 1-24.

Boldrin, Michele, Lawrence J. Christiano, and Jonas DM Fisher (2001): "Habit Persistence, Asset Returns, and the Business Cycle", American Economic Review, vol91(1), 149-166.

Bordalo, P., Gennaioli, N., Porta, R. L., and Shleifer, A. (2017): "Diagnostic Expectations and Stock Returns", Journal of Finance, forthcoming.

Bullard, J., G., Evans and S. Honkapohja (2008): "Monetary Policy, Judgment and Near-Rational Exuberance", American Economic Review, vol. 98(3), 1163-1177.

Campbell, John and John Cochrane (1999): "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior", Journal of Political Economy, vol 107, 205-251.

Carceles-Poveda, E and C. Giannitsarou (2008): "Asset Pricing with Adaptive Learning", Review of Economic Dynamics, vol.11(3), 629-651.

Carroll, C. (2003). "Macroeconomic Expectations of Households and Professional Forecasters". Quarterly Journal of Economics 118, 269 – 298.

Croce, Mariano Massimiliano (2014): "Long-run Productivity Risk: A New Hope for Production-based Asset Pricing?." Journal of Monetary Economics, vol 66, 13-31.

Coibion, O., Y. Gorodnichenko (2015): "Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts", American Economic Review, vol(105), 2644-2678.

Collin-Dufresne P., M. Johannes and L. Lochstoer (2016): "Parameter Learning in General Equilibrium: The Asset Pricing Implications", American Economic Review, vol 106(3), 664-698.

De La O, R. and S. Myers (2018). "Subjective Cash Flows and Discount Rates". Stanford University mimeo.

Ehling, P., A. Graniero and C. Heyerdahl-Larsen (2018). "Asset Prices and Portfolio Choice with Learning from Experience". Review of Economic Studies 85(3), 1752 - 1780.

Giglio S., M. Maggiori, J. Stroebel and S. Utkus (2019). "Five Facts about Beliefs and Portfolios". NBER working paper no. 25744.

Greenwood, J Robin, and A Shleifer (2014): "Expectations of returns and expected returns", Review of Financial Studies, vol 27(3) ,714-746.

Haldrup, N. and M. Jansson (2006). "Improving Size and Power in Unit Root Testing," in Palgrave Handbook of Econometrics, Volume 1: Econometric Theory, ed. by H. Hassani, T. C. Mills, and K. Paterson, Basingstoke, UK: Palgrave Macmillan, Chap. 7, 252 - 277.

Jermann, Urban J (1998): "Asset Pricing in Production Economies", Journal of Monetary Economics, vol 41(2), 257-275.

Jin, L and P. Sui (2018). "Asset Pricing with Return Extrapolation". California Institute of Technology mimeo.

Johannes, M., L. Lochstoer and Y. Mou (2016): "Learning about Consumption Dynamics", Journal of Finance, vol 71(2), 551-599.

Kuang, P. and K. Mitra (2017): "Long-run Growth Uncertainty", Journal of Monetary Economics, vol 79, 67-80.

Malmendier, U. and S. Nagel (2011): "Depression Babies: Do Macroeconomic Experiences

Affect Risk Taking? Quarterly Journal of Economics, vol126(1), 373-416.

Malmendier U. and S., Nagel (2016): "Learning from Inflation Experiences", Quarterly Journal of Economics, vol 131, 53-87.

Milani, F (2011): "Expectation Shocks and Learning as Drivers of the Business Cycle", Economic Journal, vol 121, 379-401.

Nagel, S. and Z. Xu (2018): "Asset Pricing with Fading Memory", University of Chicago mimeo.

Ng S. and P. Perron (2001). "Lag Length Selection and the Construction of Unit Root Tests with Good Size and Power". Econometrica 69, no. 6, 1519 – 1554.

Phillips, Peter CB, and Pierre Perron (1988): "Testing for A Unit Root in Time Series Regression", Biometrika, vol 75(2), 335-346.

Piazzesi, M., J. Salomao and M. Schneider (2015). "Trend and Cycle in Bond Premia." Stanford University mimeo.

Scheinkman J. and W. Xiong (2003). "Overconfidence and Speculative Bubbles." Journal of Political Economy, vol. 111, no. 6., 1183 - 1219.

Yu J. (2013): "A Sentiment-based Explanation of the Forward Premium Puzzle", Journal of Monetary Economics, vol 60, 474-491.

Appendix

A Rebasing consumption forecasts data

Since the Survey of Professional Forecasters (SPF) began, there have been a number of changes of base year in the national income and product accounts (NIPA). The forecasts for levels of consumption (SPF variable name: RCONSUM) use the base year that was in effect when the forecasters received the survey questionnaire. This Appendix explains how consumption forecasts data are rebased.

Table A0 provides the base year in effect for NIPA variables (including consumption expenditures), reproduced from Table 4 of the documentation of Survey of Professional Forecasters (p. 23). For rebasing, we use real consumption expenditures data of different vintages from the Real-Time Data Set for Macroeconomists managed by the Federal Reserve Bank of Philadelphia. Year 1996 is used as the common base year for all consumption forecast data. The data in each window needs to be rebased by multiplying a base ratio. For instance the 1959:Q4 real consumption in window from 1996:Q1 to 1999:Q3 is 1409.5 while it is 1469.5 in 1999:Q4 to 2003:Q4 window and hence the ratio is 1469.5/1409.5.

Table A0: Base years and Ratios for rebasing

Range of Survey Dates	Base Year	Ratio
1976:Q1 to 1985:Q4	1972	3.31
1986:Q1 to 1991:Q4	1982	1.48
1992:Q1 to 1995:Q4	1987	1.23
1996:Q1 to 1999:Q3	1992	1.04
1999:Q4 to 2003:Q4	1996	1
2004:Q1 to 2009:Q2	2000	0.94
2009:Q3 to 2013:Q2	2005	0.84
2013:Q3 to present	2009	0.79

B Results using mean forecasts

Table A1 and A2 test the integration properties of mean forecasts of (log) stock prices and aggregate consumption, respectively. We consider all sources of forecasts, different forecasting horizons and tests. The results suggest mean forecasts of stock prices and aggregate consumption are I(1) and not I(2) at 10% significance level.

Table A3 shows that the mean forecasts of stock prices are not cointegrated with mean forecasts of aggregate consumption with cointegrating vector $(1, -1)$ when testing at 10% significance level with one exception. That is, for 10-year ahead stock price forecasts from the Shiller survey, the null hypothesis that forecasts of 10-year ahead stock price forecast are not cointegrated with forecasts of 1-year ahead consumption with cointegrated vector $(1, -1)$ is rejected by the PP test at 10% significance level but not at 5% significance level (because the 5% critical value is -2.920).

Table A1: Integration properties: mean forecasts of $logP$

	1Q ahead	2Q ahead	4Q ahead	10-yr ahead
Panel A: I(1) test statistics				
Shiller (PP Z_t stat.)	-2.196	-2.225	-2.215	-2.400
10% critical value	-3.172	-3.172	-3.172	-3.172
Shiller (DF-GLS)	-2.218	-2.247	-2.163	-1.683
10% critical value	-2.851	-2.851	-2.851	-2.851
Livingston (PP Z_t stat.)	*n.a.*	-2.297	-2.086	*n.a.*
10% critical value	*n.a.*	-3.167	-3.167	*n.a.*
Livingston (DF-GLS)	*n.a.*	-1.698	-1.717	*n.a.*
10% critical value	*n.a.*	-2.818	-2.818	*n.a.*
Panel B: I(2) test (p-value)				
Shiller (PP Z_t stat.)	-7.576	-7.714	-7.681	-9.262
1% critical value	-2.615	-2.615	-2.615	-2.615
Shiller (DF-GLS)	-3.737	-3.724	-3.785	-3.212
1% critical value	-2.615	-2.615	-2.615	-2.615
Livingston (PP Z_t stat.)	*n.a.*	-8.506	-6.590	*n.a.*
1% critical value	*n.a.*	-2.612	-2.612	*n.a.*
Livingston (DF-GLS)	*n.a.*	-5.933	-3.154	*n.a.*
1% critical value	*n.a.*	-2.611	-2.611	*n.a.*

Table A2: Integration properties: forecasts of $logC$

	1Q ahead	2Q ahead	4Q ahead
Panel A: I(1) test			
PP (Z_t stat.)	−1.316	−1.323	−1.323
10% critical value	−3.167	−3.167	−3.167
DF-GLS	−1.225	−1.226	−1.188
10% critical value	−2.818	−2.818	−2.818
Panel B: I(2) test			
PP (Z_t stat.)	−4.696	−4.769	−4.747
1% critical value	−2.612	−2.612	−2.612
DF-GLS	−3.844	−4.006	−4.215
1% critical value	−2.611	−2.611	−2.611

Table A3: No cointegration between forecasts of $logP$ and $logC$

Mean	1Q ahead	2Q ahead	4Q ahead	10-yr ahead
I(1) test				
Shiller (PP Z_t stat.)	−2.430	−2.444	−2.389	−2.673
10% critical value	−2.595	−2.595	−2.595	−2.595
Shiller (DF-GLS)	−1.653	−1.664	−1.596	−0.917
10% critical value	−1.929	−1.929	−1.929	−1.929
Livingston (PP Z_t stat.)	*n.a.*	−2.234	−2.213	*n.a.*
10% critical value	*n.a.*	−2.591	−2.591	*n.a.*
Livingston (DF-GLS)	*n.a.*	−0.233	−0.185	*n.a.*
10% critical value	*n.a.*	−1.895	−1.895	*n.a.*

C Testing using Greenbook consumption forecasts

This Appendix shows the result of no cointegration between forecasts of stock prices and consumption still holds when we use consumption forecasts from the Greenbook data sets instead of SPF data. Our test results in the main text are robust to this alternative consumption forecast.

The Greenbook contains projections on the US economy in future quarters and is produced before each meeting of the Federal Open Market Committee. It includes projections for a large number of macroeconomic variables including real consumption growth. Four forecasting horizons are reported in each projection: 1- to 4-quarter ahead (while more horizons are issued from time to time). The dataset is published with a five-year lag. The sample of Greenbook consumption growth forecast is from 1967 to 2012. We obtain real consumption level forecast by multiplying the consumption growth forecast by (rebased) consumption level; the latter is obtained from real-time datasets for the US economy maintained by the Philadelphia Fed. To conduct the tests, we use the vintage of Greenbook forecasts in the way that the corresponding FOMC meeting date is closest to the date of the Livingston survey.

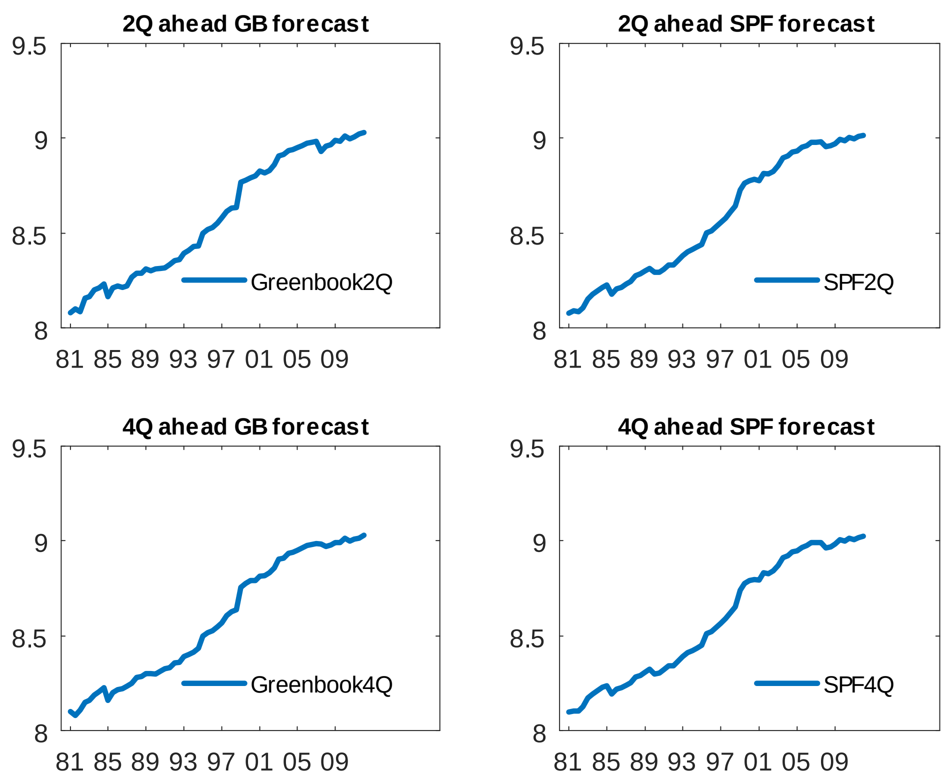

Figure A1: Greenbook and median SPF consumption forecasts

The tests are conducted using Livingston survey stock price forecasts and Greenbook consumption forecasts.[33] Figure A1 displays 2Q- and 4Q- ahead forecast of (log) consumption from the Greenbook (GB) datasets and the SPF. "Greenbook2Q" and SPF2Q"correspond to

[33]The sample period which the Shiller Survey and the Greenbook datasets overlap is relatively short. Thus, we do not conduct the test using the Shiller survey data.

the 2-Quarter ahead Greenbook and median SPF consumption forecast respectively; similarly for 4Q ahead forecast. The forecasts from the two sources look quite similar.

Table A4 reports the test statistics value and critical value for the unit root tests of forecasts of (log) aggregate consumption. For all forecasting horizons, both tests suggest that consumption forecasts is an I(1) but not I(2) process.

Table A4: Integration properties: forecasts of $logC$

	2Q ahead	4Q ahead
Panel A: I(1) test statistics		
PP (Z_t stat.)	-1.320	-1.275
10% critical value	-3.173	-3.173
DF-GLS	-1.195	-1.091
10% critical value	-2.825	-2.825
Panel B: I(2) test statistics		
PP (Z_t stat.)	-9.340	-8.416
1% critical value	-3.565	-3.565
DF-GLS	-5.469	-2.995
1% critical value	-2.615	-2.615

Table A5 shows that both PP and DF-GLS tests suggest that we cannot reject the null hypothesis that forecasts of $\log(p)$ are not cointegrated with forecasts of $\log(c)$ with cointegrating vector $(1, -1)$. This is robust to both forecasting horizons (2Q-ahead and 4Q-ahead) and using median or mean stock price forecasts.

Table A5: No cointegration between forecasts of $logP$ and $logC$

	Median 2Q ahead	Median 4Q ahead	Mean 2Q ahead	Mean 4Q ahead
I(1) test statistics				
PP (Z_t stat.)	-2.370	-2.328	-2.442	-2.321
10% critical value	-2.595	-2.595	-2.595	-2.595
DF-GLS	-0.794	-0.788	-0.808	-0.754
10% critical value	-1.903	-1.903	-1.903	-1.903

D KPSS testing results

This Appendix shows that the KPSS test rejects the null hypothesis that stock prices forecasts and consumption forecasts are cointegrated with vector $(1, -1)$. The KPSS test tests the null hypothesis of a stationary process against the alternative of a unit root. We find this test basically leads to the same conclusion as other unit root tests. Using the Livingston Survey data, the KPSS test rejects the stationarity of forecasts of stock price to consumption ratios at 1% level (see Table A6). The test results are robust to up to 11 lags for Livingston data. Using the shorter Shiller Survey data, the KPSS test rejects the null hypothesis at 5% level for 1− and 2-quarter ahead forecasts and at 10% for 4-quarter forecasts (see Table A7). The results in both tables are based on KPSS tests with one lag.

Table A6: No cointegration between median forecasts of $logP$ and $logC$

	1Q ahead	2Q ahead	4Q ahead
KPSS test			
Livingston	*n.a.*	1.43	1.57
1% critical value	*n.a.*	0.739	0.739
Shiller	0.484	0.512	0.454
5% critical value	0.463	0.463	0.463

Table A7: No cointegration between mean forecasts of $logP$ and $logC$

	1Q ahead	2Q ahead	4Q ahead
KPSS test			
Livingston	*n.a.*	1.56	1.62
1% critical value	*n.a.*	0.739	0.739
Shiller	0.487	0.465	0.428
5% critical value	0.463	0.463	0.463
10% critical value	0.347	0.347	0.347

The KPSS testing results might suffer from small sample problems. We conduct the following monte-carlo simulation to compute critical values of the KPSS test in the small sample case. We simulate the distribution of the KPSS test statistics under the null hypothesis that the long-run risks model presented in Section 4.1 is true (and hence forecasts of $\log(P/C)$ over different horizons are stationary as analyzed in the text). First, we obtain analytical expressions for 1-, 2- and 4Q ahead forecasts of $\log(P/C)$. Second, we simulate the long-run risk model with $N = 1,000$ repetitions and each sample matching the sample size and frequency of the Livingston survey data (longer than the Shiller Survey). We then get the KPSS statistics from each repetition and hence a distribution of statistics. The critical values in Table A8 are obtained from our monte-carlo simulation and differ from those in Table A6 and A7. The simulated critical values do not change much. We find that using 2- and 4-quarter ahead Livingston survey forecasts data, the KPSS test can still reject the null hypothesis of stationarity at 1% significance level. This is robust to using median or mean forecast data.

Table A8: No cointegration between forecasts of $logP$ and $logC$

	2Q ahead	4Q ahead
KPSS test		
Livingston (Median)	1.43	1.57
1% critical value	0.889	0.891
Livingston (Mean)	1.56	1.62
1% critical value	0.889	0.891

Note: Critical values based on monte-carlo simulation of the Long-run risks model