AH Premium: A Natural Experiment

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Abstract

A large proportion of Chinese twin stocks is traded in both the Shanghai (A-share) and Hong Kong (H-share) markets. A-shares have sold at a premium, known as the AH premium; this premium is large (20–50%) and volatile. AH premium provides a natural experiment by which to test asset pricing models. We show that while various standard rational expectations (RE) and Bayesian RE asset-pricing models cannot explain the AH premium, a model of internally rational learning where agents learn about stock prices can provide a natural explanation. This finding emphasizes the importance of modeling investors who learn about equity prices.

Key Words: AH Premium, Learning about Prices, Convergence Trading

JEL: G12, G15

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"The aspirin example illustrates the essential ingredients to violations of the law of one price. First, some agents have to believe falsely that there are real differences between two identical goods, and second, there have to be some impediments to prevent rational arbitrageurs from restoring the equality of prices that rationality predicts."


1 Introduction

The shares of many important Chinese companies are traded in both the Shanghai and the Hong Kong markets. Although these two sets of shares pay the same dividends and bear the same voting rights, they are distinct shares, labeled "A-shares" and "H-shares," respectively. They are different assets and cannot be traded anywhere on a one-to-one basis. The two markets were connected in November 2014, and since then, investors in either market have been able to buy both A- and H-shares of all companies participating in the connecting program. Herein, we refer to this connected market of dual-listed companies as the "AH-market".

Surprisingly, A-shares trade at a substantial premium relative to the H-shares of the same companies in the AH-market. This premium increased greatly precisely after the connection date; since then, it has been fluctuating with a mean of 27% and a maximum of 50%.1

The current study looks to explain the AH premium. Traditional asset-pricing theories—including theories based on Bayesian learning about fundamentals—have been unable to explain such a large premium. However, we find that models of learning about stock prices provide a very natural explanation for this premium.

Understanding the AH premium is important for four reasons. First, AH-market is enormous in size, and it thus has substantial implications for the global market. In 2015, the market value of AH-shares reached 2.7 trillion US dollars (USD), representing about 70% of the market capitalization of the London Stock Exchange in that year.2 Second, the premium speaks to the consequences of financial liberalization. Third, the premiums matters for investors in the AH-

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1See Figure 2.
2See section 2.2 for descriptive details of the AH-market.
market. Finally, the AH premium and the circumstances surrounding its existence provide a rare natural experiment by which economists can test asset-pricing theories in terms of stock price expectation formation.

We have not found in the literature a satisfactory explanation for the AH premium. There are some explanations based on "segmented markets" and which involve agents heterogeneous across markets. However, the AH-market has not been segmented since November 2014, and standard asset pricing theories imply that market forces should equalize prices even if market participants were heterogeneous. Other informal explanations relate to transaction costs and dividend taxes, but as we show, these would explain only a small part of the premium.\(^3\)

It is even more difficult to sustain ad-hoc explanations based on the assertion that Chinese traders are less sophisticated (i.e., "granny trading"), as most shares are owned by sufficiently wealthy investors and institutions.\(^4\) Only systematic research can discover if US or Chinese stock markets are sufficiently well described by the competitive market assumption, along with an appropriate assumption about how agents form expectations.\(^5\)

We show that learning about stock prices provides a natural explanation. Recent studies based on internal rationality (IR)—such as those of Adam and Marcet (2011), Adam, Marcet, and Nicolini (2016) (AMN), and Adam, Marcet, and Beutel (2017)—show that learning about stock prices is compatible with rational investors. These studies show that learning about prices explains the observed volatility of stock prices. Investors' subjective beliefs about stock prices are given by a model that provides a good description of actual prices; it is difficult to reject this perceived model, given actual stock prices. Under this framework, agents realize that an A-share is actually a different security from an H-share, and that it can be purchased or sold in the next period at a possibly different price. Even if the A- and H-share dividends are (nearly) identical, because investors expect A- and H-share prices to differ, those prices will actually differ in equilibrium,

\(^3\)See section 2.3 for a more detailed discussion of the literature and section 3 for a detailed analysis of how these explanations do not work for the AH premium.

\(^4\)We discuss this in section 2.2.

\(^5\)Another informal explanation has been lack of competitiveness in the Chinese stock market; this assertion, however, is difficult to square with the fact that A-shares have a higher price. In any case, lack of competition is always a possible explanation for almost anything. The US stock market is not a textbook example of competitiveness either and yet a large asset pricing literature entertains the assumption of competitive markets in American stocks.
even when investors are (internally) rational.

When agents learn about stock prices, high expected capital gains generate high prices; this situation leads to higher expected capital gains in reality, and so on, in a self-referential fashion. Our explanation for Figure 2—which presents the AH premium—would be that some factors increased either the actual prices or the expected capital gains in A-shares, or both, shortly after July 2014. (Perhaps more-liquid markets, the entry of new market participants, widespread optimistic media reports regarding the A-share market, or some special events in China around those dates led to these changes.) This increased A-prices, leading to high expected prices, and so on. We show formally that learning about prices can generate a large, volatile, and persistent AH premium, and we match the key moments of the observed AH premium data. In any case, expectation shocks to investors’ beliefs about future capital gains are compatible with the market-specific sentiment shocks proposed by Froot and Dabora (1999) to explain the difference between the prices of twin stocks.

This explanation for the AH premium implies that agents’ expectations could be a key element that determines the outcomes of financial liberalization. The average and the variation of the AH premium were relatively small a couple of years before November 2014, and the market connection was made, at least in part, to promote price convergence. As Figure 2 shows, however, the result ran counter to the aim, and the AH premium became much larger after the markets were connected. To the extent that the AH premium is considered undesirable—as it injects uncertainty into the system and may promote unproductive speculation—our observations should be relevant to the recent connection between the Shanghai and London stock markets.6

Internally rational agents’ learning about prices also matters for arbitrage. After the Shanghai and Hong Kong markets connected, the widening AH premium motivated some arbitrageurs to bet on the convergence of the A- and H-share prices, short-selling A-shares and long-buying H-shares. Chinese finance newspapers, however, reported in June 2015 that these investors ultimately

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6 Shanghai-London Stock Connect is a mechanism that connects the London Stock Exchange and the Shanghai Stock Exchange. Eligible companies listed on the two stock exchanges can issue, list and trade depositary receipts on the counterpart’s stock market in accordance with the corresponding laws and regulations. The connection took place on June 17, 2019.
encountered considerable losses.\footnote{See the news in Chinese in the link: http://finance.jrj.com.cn/2015/06/11082819343903.shtml}

The well-known limits of arbitrage discussed by Shleifer and Vishny (1997) also exist and are even more severe in Chinese markets. We show, by simulation, that on top of the well-known limits of arbitrage, arbitrageurs are still likely to incur losses, as belief-driven price differences may not diminish in the short term.\footnote{See the literature on the limits of arbitrage in section 2.3.}

The current study also contributes to the debate on how to model investors’ expectations in stock markets. As is well known, it is very difficult to explain stock price volatility and the behavior of survey expectations under rational expectations (RE). A large body of literature deviates from RE by assuming that agents imperfectly know the distribution of fundamental shocks; however, this literature assumes that investors understand the equilibrium pricing function that links stock prices to fundamentals. We dub this literature Bayesian RE,\footnote{Adam, Marcet and Beutel (2017) dub this literature Bayesian RE. They point out that this literature makes a very asymmetric assumption about fundamentals and prices: investors are assumed not to understand the behavior of fundamentals (say, dividends) but investors understand perfectly well how current stock prices relate to future fundamentals, so that investors are assumed to have RE about the pricing function.} it includes models of Bayesian learning, "agreeing to disagree", robustness and behavioral economics.\footnote{See section 2.3 on related literature for more details.} Although Bayesian RE can not explain quantitatively stock price volatility, many authors claim that it could explain survey behavior, just as learning about prices can.\footnote{See Adam, Marcet and Beutel (2017).}

Some researchers have designed lab experiments to distinguish these theories, where subjects trading stocks are informed about the dividend process and the experiment is designed to determine whether agents will quickly converge to a common knowledge of the pricing function (as is assumed in the RE and Bayesian RE literature, but not in that pertaining to learning about prices). These studies tend to show that laboratory subjects in the same group still differ in their price expectations and, accordingly, hold different positions. We contend that the AH-market tests the same issue: to the extent that traders’ stakes are many times higher in the true AH-market than in the lab, we think of this as a natural experiment, and reject RE and Bayesian RE models in favor of learning about prices.

This paper is structured as follows. Section 2 discusses institutional issues relating to the
Chinese stock market; it also reviews the literature. Section 3 argues that the standard asset pricing models where agents know the equilibrium pricing function have difficulties in generating a data-like AH premium. We show in section 4 that learning about prices can naturally explain the AH premium. Section 5 shows that convergence traders are likely to suffer considerable losses. Section 6 discusses some extensions, and section 7 discusses other possible explanations. Section 8 provides concluding remarks.

2 Overview of the Chinese Stock Markets and Literature Review

This section briefly introduces the mainland Chinese stock market, describes some key features of the AH-market, and reviews the literature that relates to our study. We point out, first, that the markets for these twin shares were segmented before November 2014, but that the AH-market was connected thereafter. We also find that it is easy to understand the AH premium before November 2014 if we use segmented market models with heterogeneous agents, as found in the literature. However, the large AH premium existing after November 2014 is a puzzle, especially given that the premium increased so much precisely after the connection.

2.1 Mainland Chinese Stock Market

The stock market in mainland China is relatively young; it opened in 1990 with the establishment of the Shanghai Stock Exchange (SSE) and the Shenzhen Stock Exchange. Stocks traded on these two exchanges that are denominated in Chinese renminbi (RMB) are called "A-shares." On average, the Chinese economy grew by 10% year over year between 1990 and 2018, and this led to the rapid development of equity markets there. The market capitalization of A-shares is the second-largest in the world, reaching approximately USD6.3 trillion at the end of 2018, with turnover of USD13 trillion in 2018.\textsuperscript{12}

Figure ?? plots the Shanghai stock price index (Shanghai Composite Index) in real terms, from

\textsuperscript{12}See data sources in Appendix 9.7.
1995 to 2018. We observe two boom–bust episodes: the SCI reached its historical peak in 2007, having risen from a trough in 2005, and then quickly busted. Stock prices boomed again in the second half of 2014 and almost doubled by mid-2015, but quickly trended downward.

Some economists and market participants have expressed the view that the Chinese central government directly and frequently intervenes in the mainland stock market. However, it is not clear whether it does so more than the US government. Publicly available information states that since 2005, the Chinese central government has intervened only once, when the A-share market burst at the end of June 2015. To preclude a severe financial crisis, the Chinese government, out of concerns about the high leverage taken by many Chinese investors, required that a national team of state-owned security companies support stock prices.\textsuperscript{13} However, it seems common worldwide for governments to intervene directly to stabilize financial market turmoil. The US government’s 2008 intervention (e.g., the Troubled Asset Relief Program) is a prominent example (Veronesi and Zingales 2010). The intervention of the Hong Kong government in the stock market during the 1998 Asian financial crisis is also well documented (Goodhart and Lu 2003).

\subsection*{2.2 \textbf{AH-shares and Shanghai-Hong Kong Stock Connect}}

The shares of companies incorporated in mainland China but traded on the Hong Kong Stock Exchange (HKSE) in Hong Kong dollars (HKD) are called H-shares. Currently, about 100 mainland companies dual-list in both the A-share (mostly on the SSE) and H-share markets.\textsuperscript{14} Twin shares of the same companies have the same fundamentals, as they are identical with respect to shareholder rights such as voting and profit-sharing. Information clarifying that the two classes of shares entertaining the same fundamental is widely available. However, they are different stocks because they have different international securities identification numbers, which serve in the standardized identification of securities. For instance, the A-share of Air China is identified as CNE000001NN0, but its H-share is identified as CNE1000001S0. It is impossible to sell one class of shares (e.g., CNE000001NN0) in the market of another class of shares (e.g., CNE1000001S0).

Many Chinese companies with large market capitalization are included in the AH-market:

\textsuperscript{13}See Huang, Miao and Wang (2019).

\textsuperscript{14}Only a few companies list in SZSE. Shenzhen and Hong Kong markets also connected from December 2016.
the market value of the A-shares of these dual-listed companies accounts for 20% of all A-share market capitalization, while the market value of the H-shares of these companies accounts for 35% of HKSE market capitalization.\textsuperscript{15}

Before November 17, 2014, the Shanghai and Hong Kong markets were segmented; mainland investors were not allowed to trade in the H-share market, and Hong Kong and international investors were barred from investing in the A-share market. Therefore, any AH premium prior to November 2014 can be explained by appealing to segmented markets, along with heterogeneous agents. However, the Shanghai and Hong Kong markets were connected after the Shanghai–Hong Kong Stock Connect program was initialized on November 17, 2014.\textsuperscript{16}

Through the Connect program, mainland investors can trade more than 300 stocks in the HKSE, and foreign investors can trade more than 500 stocks in the SSE (Ma, Rogers and Zhou 2019). All Shanghai and Hong Kong twin stocks are part of the Connect program. Mainland investors can participate in the H-share market (southbound trading) very easily, through the SSE’s trading and clearing facilities; Hong Kong and international investors, meanwhile, are allowed to trade in the Shanghai A-share market (northbound trading).\textsuperscript{17} In each market, transaction costs are low and identical for local and foreign investors alike; we discuss these matters in section 7. The trading and settlement currency of the Shanghai–Hong Kong Stock Connect program is the RMB; mainland investors do not need HKD to buy H-shares via the Connect program. The China Securities Depository and Clearing Company provides unlimited currency conversion service, automatically and in the background, for mainland investors trading in the H-share market. After mainland investors sell H-share stocks, they obtain proceeds in RMB rather than HKD. Meanwhile, Hong Kong and international investors can acquire RMB easily in the Hong Kong offshore RMB market to buy A-shares.\textsuperscript{18}

\textsuperscript{15}Refer to Appendix 9.7 for data sources. Also see an analysis artical by the famous investment bank economist Shanwen Gao. (http://www.thfr.com.cn/post.php?id=34627)


\textsuperscript{17}Precisely, all Hong Kong and international investors are allowed to trade A-shares listed in Shanghai. All mainland institutional investors and sufficiently wealthy individual investors who have at least RMB 500,000 in their accounts are eligible to trade H-shares. According to the Financial Times report in July 2015, small retail investors hold less than 5% of the overall market value of A-shares (https://www.ft.com/content/f3d94f92-2715-11e5-9e4e-a775d2b173ca).

\textsuperscript{18}Northbound trading and Southbound trading are respectively subject to a separate set of Daily Quota. The
Surprisingly, the AH premium increased markedly after the connection date. The Hang Seng China AH Premium Index plotted in Figure ?? measures the weighted average price ratio (i.e., A-share prices over H-share prices of the same companies) in percentage for these AH-twin shares.\(^{19}\)

An index value of 100 means that A-shares are trading at par, while an index value exceeding 100 indicates that A-shares are trading at a premium. After the market connection, according to standard asset-pricing theory, the AH premium index should have converged to 100 (or to somewhere close to 100, taking into account the different dividend taxes and transaction costs). However, it diverged dramatically, to a peak value of almost 150 in June 2015; since then, it has fluctuated between 120 and 150. Note that the AH premium index had been increasing in the four months prior to the connection, and that the rising momentum persisted before it was mean-reverted. As for individual stocks, most had a larger premium for the A-shares after the connection. Hence, the AH premium does not derive solely from a few market-weight-dominating companies.\(^{20}\)

A- and H-share prices highly and positively correlate, as shown in Figure ??: from November 2014 to June 2015, both A- and H-share prices increased, but the A-share prices increased more quickly, contributing to the divergence in prices. We can see a similar pattern of different speeds of adjustment when the price indexes fell after July 2015.

Note: A(H) is the price index of A-shares (H-shares) of the dual listed companies.

### 2.3 Related Literature

This paper is related to the literature on the price differences of twin stocks. Many studies work to explain the price differences in the segmented market environment, where AB premium is a prominent example.\(^{21}\) Heterogeneities among different participants across segmented markets—

\(^{19}\)Let \(P^A\) be the weighted average price of A-shares and \(P^H\) the weighted average price of H-shares, the index is \(P^A/P^H \times 100\). The exchange rate is taken into account when constructing this index. The index methodology refers to https://www.hsi.com.hk/static/uploads/contents/en/dl_centre/methodologies/IM_chinaahe.pdf


\(^{21}\)see Appendix 9.1 and Mei, Scheinkman and Xiong (2009) for details of AB premium.
such as agents’ different stochastic discount factors (Fernald and Rogers 2002), and diverse beliefs about fundamentals (Chan, Menkveld and Yang 2008; Mei, Scheinkman and Xiong 2009; Jia, Wang and Xiong 2017) are argued to explain the price differences. However, heterogeneities fail to explain price differences that existed after the November 2014 market connection.

The literature also widely documents the difficulties of explaining price differences in connected markets. For instance, Froot and Dabora (1999) show that price differences in three twin stocks in connected U.S. and European markets are affected by the location of trade and that transaction costs are not the main reason.\footnote{They are Royal Dutch Petroleum and Shell Transport and Trading, PLC; Unilever N.V. and Unilever PLC; and SmithKline Beecham.} Lamont and Thaler (2003) consider this an anomaly in violation of the law of one price, and that it is difficult to rationalize.\footnote{They think that one partial answer to the premium of Royal Dutch over Shell is that Royal Dutch was a member of the S&P 500 index, but Shell was not.} The AH premium is even more puzzling, given the much larger market size relative to those discussed in the aforementioned studies. The exogenous "nonfundamental" demand shocks of irrational noise traders—which can drive stock prices away from their fundamental values, as in De Long et al. (1990), Gromb and Vayanos (2010), and others—also have the potential to produce price differences; however, these studies do not explicitly address this issue. To the best of our knowledge, the current study is the first to propose a microfounded asset-pricing model with rational agents to rationalize and quantitatively explain the premium in twin stocks and, in particular, the AH premium.

As is well known, external habit (Campbell and Cochrane 1999) and long-run risk (Bansal and Yaron 2004) can explain stock market regularities under RE; nonetheless, some difficulties inherent in these theories have been highlighted. For example, these theories cannot in any case explain survey behavior. An extensive body of literature deviates from RE by assuming that agents imperfectly know the fundamentals, but this Bayesian RE literature assumes that investors, like investors in the RE literature, know the equilibrium pricing function, or the mapping from fundamentals to prices. It includes models of Bayesian learning (e.g., Timmermann 1996; Collin-Dufresne, Johannes and Lochstoer 2016), agreeing to disagree (e.g., Scheinkman and Xiong 2003; Ehling, Graniero and Heyerdahl-Larsen 2018), robustness (e.g., Cogley and Sargent 2008), and behavioral economics (e.g., Barberis, Shleifer and Vishny 1998; Barberis, Greenwood, Jin and
However, these studies cannot explain the AH premium, since the equilibrium prices of stocks paying the same dividends are equal, even in the presence of heterogeneous agents. Recent studies propose various strategies by which to test these asset-pricing theories. Greenwood and Shleifer (2014), Adam, Marcet, and Beutel (2017), and Kuang, Zhang, and Zhang (2019) use survey expectations data to test expectation formation in multiple asset-pricing models. Bansal, Gallant, and Tauchen (2007), Beeler and Campbell (2012), Bansal, Kiku, and Yaron (2012), and Barro and Jin (2016) apply moment-matching methods to compare the empirical performance of habit, long-run risk, and rare disasters models. Aldrich and Gallant (2011) use a Bayesian framework to compare habit, long-run risk, and prospect theory. We instead use the AH premium in connected markets as a natural experiment to test directly various asset-pricing models in terms of expectation formation.

To rationalize the AH premium in connected markets (and price differences in twin stocks in general), it may be necessary to relax the assumption of agents’ knowledge of the equilibrium pricing function; this is done in various adaptive learning models. Examples of this type of model are those of Lansing (2010), Branch and Evans (2011), Boswijk, Hommes, and Manza (2007), and Carceles-Poveda and Giannitsarou (2008), all of whom built asset-pricing models where agents learn adaptively and do not have perfect knowledge about the true stochastic process for payoff-relevant variables beyond their control. Adam and Marcet (2011) developed IR to provide microfoundations for adaptive-learning models. Adam, Kuang, and Marcet (2012) show an application to housing prices, and Adam, Marcet, and Nicolini (2016) explain stock price volatility. Winkler (2019) matches both asset price and business cycle moments.

The experimental literature also provides evidence supporting the presence of subjective price beliefs. Hirota and Sunder (2007) and Asparouhova et al. (2016) designed a Lucas asset-pricing laboratory experiment, and they found that stock prices display excess volatility unaccounted for by fundamentals; they assert that the volatility can most likely be attributed to participants’ subjective price expectations. Crockett et al. (2018) suggest that speculative behavior in the lab causes the emergence of price bubbles.

Finally, our study relates to the literature on arbitrage limits. Shleifer and Vishny (1997)
highlight important arbitrage limits in the real world, and think that arbitrage limits are why arbitrage fails to eliminate a number of anomalies in financial markets. Xiong (2001) offers a formal model on the convergence trading strategy, and he finds that convergence traders amplify unfavorable shock by liquidating their positions when the shock of noise traders causes them to suffer substantial capital losses. In the work of De Jong, Rosenthal, and Van Dijk (2009), the uncertainty that convergence traders face arises from the absence of an identifiable date at which dual-listed stock prices will converge. Our results are in line with those of these studies. We posit that similar arbitrage limits also exist in the Chinese market. On top of the well-known limits of arbitrage, we find that learning about prices (caused by internally rational agents rather than by noise traders) also hinders arbitrage.

3 AH Premium with Knowledge of Pricing Function

To date, there has been no theoretical study of the features that can generate a premium in twin stocks. In this subsection, we discuss various models by which to determine whether available theories where agents know the equilibrium pricing function (that is, RE or Bayesian RE) can explain the observations. From this point, the assumptions are that after November 2014, the AH-market is not segmented and traders in both markets can purchase stocks in the other market. Throughout this subsection, we focus on an economy lacking rational bubbles.\footnote{We discuss the rational bubbles in section 7.}

We argue that it is challenging for the standard RE and Bayesian RE paradigm to explain the premium in connected markets. Essentially, knowledge of the equilibrium pricing function places a strict restriction on agents’ price expectations. A- and H-share prices are the same in equilibrium as agents expect them to be.

It is clear that a homogeneous agent model with RE or Bayesian RE cannot explain the premium in twin stocks, as the agent is marginal for both shares in each period and the twin shares enjoy the same payoff in each state. However, it might be less clear how heterogeneous agents in the standard RE and Bayesian RE paradigm relate to the premium in connected markets. Some authors argue that heterogeneities across agents can explain price differences of twin shares.
across segmented markets. Heterogeneity can be due to features such as different SDFs (Fernald and Rogers 2002) and diverse beliefs about fundamentals (Chan, Menkveld and Yang 2008; Mei, Scheinkman and Xiong 2009; Jia, Wang and Xiong 2017) between mainland investors and foreign investors. In addition, informal explanations of the AH premium in connected markets are still often based solely on heterogeneity. 25

We find it useful to explain that heterogeneity with RE or Bayesian RE does not generate a premium in twin shares bearing the same fundamentals. It can explain only different consumption allocations and bond positions: it cannot explain price differences. We illustrate this with several models for which there is a well-developed theory and where we can use well-known results. The common setup for all models shown in subsection 3.1 is that the economy is populated with two types of infinitely lived agents, with type 1 agents standing for mainland investors and type 2 agents for Hong Kong and international investors. Within each group, the agents are homogeneous, with the same preference satisfying Inada conditions. As is common in the RE and Bayesian RE literature, agents understand the equilibrium pricing function mapping from fundamentals to prices, but agents may understand fundamentals imperfectly. Moreover, it is common knowledge that A- and H-shares pay the same dividends. All agents can participate in the market of each share, but are confronted with short-selling constraints for stocks. Finally, Ponzi schemes and rational bubbles are ruled out.

The literature suggests that transaction costs levied on trading values can explain only a very small part of the premium in twin stocks; this holds for the AH premium case. We also find that dividend taxes can bring about some price differences. In subsection 3.2 we consider dividend taxes by embedding them in the models discussed in subsection 3.1, and we argue that they cannot quantitatively explain the observed AH premium.

3.1 Models with Pricing Knowledge

This section presents results that follow directly from the literature on heterogeneous agents and "agreeing to disagree." We first show analytically how heterogeneity in RE and Bayesian RE fails to produce any price difference in complete markets with two types of agents. The two types of agents differ in discount factors and utilities in the first model and in beliefs about fundamentals in the second. These two simple models carry the basic logic and have analytical expressions.

In the first case, type \(i\) agents of \(\mu^i\) fraction have utility \(u^i\) and a subjective discount factor \(\delta^i\), for \(i \in \{1, 2\}\). Preference \(u^i\) satisfies the Inada condition.\(^{26}\) Investors consume goods \(C^i_t\), receive a deterministic endowment \(Y_t\) and choose portfolios.\(^{27}\) Investor portfolios consist of A-shares \(S^i_{t, A}\), H-shares \(S^i_{t, H}\), and Arrow securities (state-contingent bonds) \(B^i_t(D)\). The two classes of equities share the same dividend payments \((i.e. D^A_t = D^H_t = D_t)\). The exogenous process of dividend \(D_t\) is assumed to be i.i.d with probability \(\text{prob}(D)\) taking values of \(D \in \mathcal{D}\) over the set of states \(\mathcal{D}\) and investors know this perfectly. The state-contingent bonds \(B_t(D)\) purchased in period \(t\) deliver one unit of consumption if the realized dividend in period \(t + 1\) is \(D\).

The aggregate supply of each share is normalized to 1, such that the market-clearing conditions of equities read as

\[
\mu^1 S^1_{t,j} + \mu^2 S^2_{t,j} = 1, \quad j \in \{A, H\}.
\]

The market-clearing condition of A-shares distinguishes it from H-shares, thus indicating that the twin shares are distinct assets. Recall that it is forbidden to sell A-shares in the H-share market and vice versa.

The state-contingent bond market clears with

\[
\mu^1 B^1_t(D) + \mu^2 B^2_t(D) = 0 \quad \forall D \in \mathcal{D}.
\]

\(C_t\) is the aggregate consumption supply. The feasibility constraint \(C_t = Y_t + 2D_t = \mu^1 C^1_t + \mu^1 C^2_t\) is satisfied by Walras’s law.

\(^{26}\)Preference can be CRRA utility, habit utility or Epstein-Zin utility.

\(^{27}\)\(Y_t\) is assumed to be deterministic without loss of generality, it is straightforward to extend it to the case with stochastic \(Y_t\).
Type $i$ agents maximize expected lifetime utility

$$\max_{\mathbf{C_t}} \sum_{t=0}^{\infty} E_t(\delta^t) u^i(C_t^i)$$

subject to the budget constraint

$$S_t^i A_t + S_t^i H_t + C_t^i + \sum_{D \in D} B_t^i(D) Q_t(D)$$

$$= S_t^{i,A}_{t-1}(P_t^A + D_t) + S_t^{i,H}_{t-1}(P_t^H + D_t) + B_t^{i}_{t-1}(D_t) + Y_t, \forall t$$

and bounds on shareholding

$$S \leq S_t^{A,i} \leq \mathbb{S} \quad (1)$$

$$S \leq S_t^{H,i} \leq \mathbb{S}.$$  

Type $i$ agents in period $t$ consume $C_t^i$ amount of goods, purchase A-shares $S_t^{i,A}$ at price $P_t^A$, purchase H-shares $S_t^{i,H}$ at price $P_t^H$, purchase Arrow securities $B_t^i(D)$ at price $Q_t(D)$, and receive payments from the outstanding shares $S_t^{i,A}_{t-1}(P_t^A + D_t)$ and state-contingent bonds $B_t^{i}_{t-1}(D_t)$ as well as the endowment $Y_t$. The upper and lower bounds serve to preclude a Ponzi scheme and will be common in the remainder of the paper.

The standard optimal full-insurance conditions in equilibrium read as

$$\delta^1 u_c^1(C_{t+1}^1) = \delta^2 u_c^2(C_{t+1}^2) \quad \forall D \in D$$

(2)

where $u^i_c$ is the marginal utility with respect to $C_t^i$. The SDFs of the two types of agents are identical in each state. This condition implies that there exists a unique market SDF. Since agents are assumed to know the pricing mapping from the fundamentals to prices—namely, the present-value form of equity prices—we have the following analytical expression using type $i$ agent’s SDF

\[28\text{As is usually assumed in the literature, the lower bound is more stringent than the upper bound.} \]
\[ P_t^A = P_t^H = E_t \sum_{j=1}^{\infty} (\delta^i)^j \frac{u_i^j(C_t^{i+j})}{u_e(C_t^i)} D_{t+j} \]

This equation clearly delivers zero AH premium, even with heterogeneous agents.

Besides heterogeneities in preferences, investors can differ in their beliefs about fundamentals. Some researchers document that foreign investors tend to be pessimistic about the Chinese economy. In contrast, mainland Chinese citizens have more optimistic views (e.g., Jia, Wang and Xiong 2017) or have an information advantage over foreigners about fundamentals (e.g., Chan, Menkveld and Yang 2008). Some stockbrokers and market analysts tend to propagate this kind of story as a way to rationalize the AH premium.

To analyze the potential explanatory power of this explanation, we turn to the second case of Bayesian RE, in which agents do not know the objective probability \( \text{prob}(D) \) of the dividend. Specifically, we assume type \( i \) agents have subjective beliefs about dividend \( \text{Prob}^i(D) \). This is the only difference from the first case. These heterogeneous beliefs are common knowledge for all agents, but they agree to disagree. To isolate the effects of diverse beliefs, we abstract away heterogeneous preferences.

In this case, the first-order conditions with respect to the contingent bonds now read as

\[ \delta \frac{u_e(C_{t+1}^1(D))}{u_e(C_t^1)} \text{Prob}^1(D) = \delta \frac{u_e(C_{t+1}^2(D))}{u_e(C_t^2)} \text{Prob}^2(D) \forall D \in \mathcal{D} \tag{3} \]

Although the agents’ SDFs differ on account of their heterogeneous beliefs concerning fundamentals, the subjective probability adjusted SDFs are identical in each state. Equation (3) immediately implies that the prices evaluated with agent 1’s SDF and subjective probability are equal to those evaluated with agent 2’s SDF and subjective probability; the conclusion of no premium still applies—that is to say,
\begin{align*}
P_t^A &= P_t^H = E_t^1 \left[ \sum_{j=1}^{\infty} (\delta)^j \frac{u_c(C^1_{t+j})}{u_c(C^1_t)} D_{t+j} \right] \\
&= E_t^2 \left[ \sum_{j=1}^{\infty} (\delta)^j \frac{u_c(C^2_{t+j})}{u_c(C^2_t)} D_{t+j} \right].
\end{align*}

This equilibrium pricing function dictates that stock prices are determined by fundamentals in the equilibrium. The prices of twin shares will be the same as a result of having the same fundamentals along with the knowledge of the pricing function.

Although heterogeneity cannot produce the AH premium, it does affect the allocation of consumption and bond positions. For instance, having higher risk aversion induces smoother consumption; agents with a larger subjective discount factor and more precise subjective beliefs about fundamentals will accumulate assets and achieve dominance in the economy eventually. A simulation-based illustration of this is found in Appendix 9.2.

These models involving the Arrow securities serve to provide the basic logic for why heterogeneities across investors are not relevant to the AH premium in connected markets. However, despite the latest ongoing innovations in financial and insurance markets, real-world markets remain arguably incomplete. We want to show that the conclusion of no premium carries over to the incomplete market environment when agents have pricing knowledge. The following example illustrates this.

Let us turn to a variant of the model discussed above, but without access to the Arrow securities. For the incomplete market, consider a typical process for the endowment \( Y_t \) and dividend payment \( D_t \), as in Campbell and Cochrane (1999). This setup will also be valid in the learning model in section 4. Specifically, we directly impose restrictions on the dividend and the aggregate consumption supply processes:

\begin{align*}
\frac{D_t}{D_{t-1}} &= a\epsilon^d_t, \log \epsilon^d_t \sim i.i.d. N\left(-\frac{s^2_d}{2}, s^2_d\right) \quad (4) \\
\frac{C_t}{C_{t-1}} &= a\epsilon^c_t, \log \epsilon^c_t \sim i.i.d. N\left(-\frac{s^2_c}{2}, s^2_c\right) \quad (5)
\end{align*}
The dividend and consumption growth rates share the same mean \(a\), and \((\log \epsilon^d_t, \log \epsilon^c_t)\) are jointly normally distributed with a correlation of \(\rho_{c,d}\), and the standard deviations of \(s_d\) and \(s_c\). The endowment \(Y_t\) in the economy is given by the feasibility constraint \(C_t = Y_t + 2D_t\).

Agents might have imperfect knowledge about (4) and (5). They maximize their expected lifetime utility subject to the budget constraint with no access to the Arrow securities

\[
S^{i,A}_t P^A_t + S^{i,H}_t P^H_t + C^i_t
\]

as well as the shareholding constraints (1).

In this case, the twin shares are perfect substitutes for any agent as they realize that the twin shares delivering the same payments in each state play an identical role in the economy and expect the twin shares to have the same prices through the equilibrium pricing function.

In general, we provide the following argument for a lack of premium in the incomplete market models when agents know the equilibrium pricing function. Although marginal persons can switch across the two types of agents over time, the present-value formula of prices still holds, as agents know the pricing function.\(^{29}\) Therefore, there exists one equilibrium where the prices of twin shares equal the present value of dividends, and prices are equal.\(^{30}\)

The intuition is that the prices are the same in equilibrium, as each agent expects them to be as per knowledge of the equilibrium pricing function. Knowledge of the equilibrium pricing function places a strict restriction on agents’ expectations about prices. Agents’ price expectations are governed by their expectations about fundamentals through the equilibrium pricing function: there is no room for investors to entertain different expected prices across the twin shares to yield price differences. When agents know the equilibrium pricing function, heterogeneity among agents is irrelevant.

---

\(^{29}\)With the common knowledge about each agent’s preference and belief, agents know who is the marginal person in the current period and have the right expectations about who are going to be the marginal persons in the future.

\(^{30}\)It is well known that there exists an SDF that price all payoffs in the incomplete market. Refer to Ross (1978) and Luttmer (1996).
As is discussed below, dividend taxes resulting in some differences in the fundamentals across the two classes of shares are key to producing price differences in the RE and Bayesian RE paradigm. However, they are not sufficiently large to explain the premium quantitatively.

### 3.2 Dividend Taxes

The real world is not free of dividend taxes, transaction costs, and exchange rate risk. Here, we highlight dividend taxes; we defer the discussion of transaction costs and exchange rate risk to section 7, as they appear not to be as remarkable as dividend taxes.

According to the stipulated financial regulations, mainland investors pay a constant 20% dividend tax rate for holding H-shares, while Hong Kong and international investors pay a 10% dividend tax for either share type. Dividend taxes for mainland investors to hold A-shares depend on the holding period. The dividend tax rate is 20%, 10%, and 5% for investors holding the shares for less than one month, between one month and one year, and more than one year, respectively.\(^{31}\)

Let \(\tau_{i,A} \) and \(\tau_{i,H} \) represent the dividend taxes confronted by type \(i\) agents in the A- and H-share markets, respectively. For type 1 agents, the after-tax dividend payments per share are \((1 - \tau_{1,A})D_t\) for A-shares and \((1 - \tau_{1,H})D_t\) for H-shares; while for type 2 agents they are \((1 - \tau_{2,A})D_t\) and \((1 - \tau_{2,H})D_t\), respectively, where \(\tau_{1,H} = 20\%\), \(\tau_{2,A} = \tau_{2,H} = 10\%\). In particular, we deliberately choose the 5% dividend tax for type 1 agents to hold A-shares (i.e. \(\tau_{1,A} = 5\%\)) to give the model below the best chance to match the high AH premium, as it will becomes later that this induces the largest wedge between dividend taxes confronting the marginal persons across the two markets. Given these dividend taxes, type 1 agents have more after-tax dividend from A-shares, while type 2 agents obtain more after-tax dividend from H-shares (i.e. \((1 - \tau_{1,A})D_t > (1 - \tau_{2,A})D_t\) and \((1 - \tau_{2,H})D_t > (1 - \tau_{1,H})D_t\)).

We explore the effects of dividend taxes on the price premium by incorporating them into the models discussed in subsection 3.1. Although different dividend tax rates across the two markets can give rise to some price differences, we argue that they cannot explain some basic features of the observed AH premium.

\(^{31}\)Refer to Appendix 9.7 for data sources pertaining to transaction costs and dividend taxes.
To begin with, we embed dividend taxes in the complete market RE model in subsection 3.1. One can easily prove that, with the dividend tax scheme, an equilibrium is as follows: agent 1(2) buys all A(H)-shares in the first period, and they are against the short-selling constraint on the other shares (corner solutions); each agent holds their position forever. This is the equilibrium because of the short-selling constraint and the fact that both agents have the same SDF (due to complete markets). Due to the tax advantage of agent 1(2) for A(H)-shares, it is agent 1(2) who is marginal for A-shares (H-shares). Therefore, we have

\[
P^A_t = E_t \sum_{j=1}^{\infty} (\delta^1)^j \frac{u^1_c(C^1_{t+j})}{u^1_c(C^1_t)} (1 - \tau^{1,A}) D_{t+j} \tag{7}
\]

\[
P^H_t = E_t \sum_{j=1}^{\infty} (\delta^2)^j \frac{u^2_c(C^2_{t+j})}{u^2_c(C^2_t)} (1 - \tau^{2,H}) D_{t+j}. \tag{8}
\]

Given that the SDFs of the two types of agents are identical and the constant \((1 - \tau^{1,A})\) and \((1 - \tau^{2,H})\) can be factored out, we derive

\[
\frac{P^A_t}{P^H_t} = \frac{1 - \tau^{1,A}}{1 - \tau^{2,H}}. \tag{9}
\]

Therefore, price differences are driven by different dividend tax rates confronting the marginal agents across the two markets. This price ratio is constant over time at 105.6%; this is inconsistent with the size and fluctuation of the observed AH premium.\(^{32}\) This equation still holds when agents have heterogeneous beliefs on dividends, because of the equation (3).

Thus far, the argument in this subsection relies on the existence of state-contingent bonds. Let now us consider the case without access to state-contingent bonds. In this case, agent type 1(2) will hold H(A)-shares with positive probability—in other words, the agents may invest in both markets in order to accumulate precautionary savings. The exact dynamics of this incomplete market model will depend precisely on the processes for income, the type of heterogeneity, the assets available, and the like. However, in many setups, in the presence of a dividend tax advantage,

\(^{32}\)One can show that \textit{ceteris paribus}, there would be no premium if \(\tau^{1,A} = 10\%\) or 20\%.

22
it will still be the case that with probability 1 agent 1(2) holds A(H)-shares in all periods—that is, each agent is marginal in "their" market with probability 1, against the short-selling constraint in the "other" market for most periods, although they may also be marginal in the "other" market in some periods. Under these circumstances, equations (7) or (8) still hold, since the Euler equation for the "own" market always holds.

Now, as is well known, SDFs are not equal across agents under incomplete markets in all periods, and so we cannot derive (9) exactly as we did before. However, many studies within the incomplete-market literature report that equilibrium prices are set so as to make SDFs across agents almost equal in incomplete market equilibria; we therefore have $\frac{u_c(C_{1t+1}^1)}{u_c(C_{1t}^1)} \approx \frac{u_c(C_{1t+1}^2)}{u_c(C_{1t}^2)}$. Combining these two observations we have that (9) holds approximately around the steady state with corner solutions. However, as we argued above, (9) is so far from explaining the AH premium.

Note: $a_1$ is type 1 agents’ holding of A-shares, $a_2$ is type 2’s holding of A-shares, $h_1$ is type 1’s holding of H-shares and $h_2$ is type 2’s holding of H-shares.

Along with the previous argument, we also simulate a calibrated transitional economy where both shares are split equally among the two agents in the initial period; we assume the two types of agents have the same fractions (i.e., $\mu^1 = \mu^2 = 0.5$) and that agents actively trade shares with each other afterwards. We incorporate dividend taxes into previous incomplete market model in subsection 3.1. We assume each agent has the same logarithm utility to isolate the effects of dividend taxes. For simplicity, we also take away endowment $Y_t$. Calibration for the dividend process is the same as that in the following section 5.

We find that over the transition, both agents are marginal for A- and H-shares, and assets are traded in equilibrium such that the twin shares have the same prices. The dynamics of

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33 An early paper reporting a high correlation of SDFs in a two agent model with two assets and heterogeneous incomes is Marcet and Singleton (1999) (see their Table 2 reporting a high correlation of individual consumption and aggregate income for calibrations of Models 1-3 with high serial correlation of income). Levine and Zame (2001) argue that for high discount factors incomplete market equilibria nearly completes the markets. In large OLG models young agents are against a borrowing constraint for a long time so that model their SDF is not highly correlated with that of old agents, but large differences in age do not seem to be an issue in the AH-market. Some recent models argue that different liquidity can also drive a wedge between agents’ SDFs. We do not address issues of liquidity in this paper except to point out that liquidity is very high in both A- and H- markets, see our comment in section 7.

34 The simulation method follows Marcet and Singleton (1999).
share-holding, as in Figure ??, show that over the transition, type 1 agents accumulate A-shares and type 2 agents accumulate H-shares. This is because agent 1(2) has a tax advantage with A(H)-shares. However, we find no price differences. Therefore, we conclude that the observed premium is incompatible with incomplete markets, as the price differences are close to (9) or even lower when agents actively trade with each other over the transition.

4 An Internal Rationality Learning Model

Our conclusion from section 3 is that staying within the standard paradigm of agents who know the equilibrium pricing function cannot suffice in quantitatively explaining the observed behavior of the AH premium. In this section we show that a parsimonious model involving "internal rationality" that learns about prices—in which rational agents do not know the equilibrium pricing function—can quantitatively explain the AH premium. As Adam and Marcet (2011) point out, when agents’ preferences and beliefs are not common knowledge, agents cannot deduce the equilibrium pricing function from their own optimization conditions;\textsuperscript{35} additionally, their subjective expectations about future prices are not anchored by their expectations about fundamentals, and their expectations about prices become crucial in pricing an equity.

Agents have subjective beliefs about prices. Investors’ subjective beliefs about stock prices are described by a model that provides a good description of actual prices. It is difficult to reject this perceived model, given actual stock prices. Under this framework, agents realize that an A-share is actually a different security from an H-share, and that it can be purchased or sold in the next period possibly at a different price. Agents learn about prices and, in this way, a difference in expected prices across the twin shares feeds on itself to generate the AH premium. Even if the A- and H-dividends are (nearly) the same, the A- and H-prices will actually differ in equilibrium, simply because investors expect them to differ. This behavior is compatible with rationality.

\textsuperscript{35}In Adam, Marcet and Niconoli (2016) and Adam, Marcet and Beutel (2017), despite agents are homogeneous, homogeneity is not the common knowledge. Each agent probably believes that others have very different beliefs from them.
4.1 Model Environment

The environment is almost identical to that in the incomplete market case in subsection 3.1. The consumption process and dividend process (4) and (5) still hold. Agents face the budget constraint (6) and shareholding constraints (1).

The economy is populated by a unit mass of infinite-horizon agents who are homogeneous. This homogeneity, however, is not common knowledge among agents, and in this respect this model differs from previous models. Here, we abstract from the agents heterogeneity to isolate the role of learning about prices in generating a high, volatile, and persistent AH premium.\textsuperscript{36}

Agents’ preference is specified with the standard constant relative risk aversion utility with a relative risk aversion of $\gamma$ for quantitative exercise

$$E_P^0 \sum_{t=0}^{\infty} \delta^t \frac{(C_t^i)^{1-\gamma}}{1-\gamma}.$$  

Importantly, instead of the objective probability measure, expectations are formed using the subjective probability measure $\mathcal{P}$, which describes the probability distributions for all "external" variables. Subsection 4.2 contains more details about the probability space.

4.2 Probability Space

In this subsection we explicitly describe the general joint-probability space of the variables that agents take as given ("external" variables). Stock prices $\{P_t^A, P_t^H\}$ under a competitive stock market assumption, and exogenous endowment and dividend processes $\{Y_t, D_t\}$ are external to the agents’ decision problem. In section 3, agents know the equilibrium pricing function (as in RE and Bayesian RE models), and so stock prices carry only redundant information. However, when agents do not know the equilibrium pricing function (as a result of having no common knowledge about agents’ preferences and beliefs), equilibrium stock prices should be included in the underlying state space of the agents’ decision problem. Formally, we define the probability space as $(\mathcal{P}, \mathcal{B}, \Omega)$ with $\mathcal{B}$ denoting the corresponding $\sigma$-Algebra of the Borel subsets of $\Omega$ and $\mathcal{P}$

\textsuperscript{36}Heterogeneity may be an interesting way to complement the main story of this section. But we think learning about prices is the main story.
denoting the agents’ subjective probability measure over \((B, \Omega)\). The state space \(\Omega\) for the realized exogenous variables is

\[ \Omega = \Omega_Y \times \Omega_D \times \Omega_{PA} \times \Omega_{PH} \]

where \(\Omega_X\) represents the state space for all possible infinite sequences of the variable \(X \in \{Y, D, P^A, P^H\}\). Thereby, a specific element in the set \(\Omega\) represents an infinite sequence \(\omega = \{Y_t, D_t, P^A_t, P^H_t\}_{t=0}^{\infty}\). Agent \(i\) chooses plans for the endogenous variables \(C^i_t, S^{Ai}_t, S^{Hi}_t\) contingent on future realizations of \(\Omega^t\), where \(\Omega^t\) represents the set of histories from period zero up to period \(t\)—that is

\[ (C^i_t, S^{Ai}_t, S^{Hi}_t): \Omega^t \rightarrow \mathbb{R}^3. \]

The expected utility with the probability measure \(\mathcal{P}\) is defined as

\[
E^P_0 \sum_{t=0}^{\infty} \delta^t \frac{(C^i_t)^{1-\gamma}}{1-\gamma} \equiv \int_{\Omega} \sum_{t=0}^{\infty} \delta^t \frac{C^i_t(\omega^t)^{1-\gamma}}{1-\gamma} dP(\omega^t).
\]

We assume agents know the processes of dividend and consumption supply (4) (5) to differentiate from Bayesian RE. Subsection 4.4 specifies the subjective probability distribution for prices.

### 4.3 Optimality Conditions

Since the objective function is concave and the feasible set is convex, the agent’s optimal plan is characterized by the first–order conditions

\[ (C^i_t)^{-\gamma} P^i_t = \delta E^P_t ((C^i_{t+1})^{-\gamma}(P^i_{t+1} + D_{t+1})) \quad j \in \{A, H\} \]

(10)

Using standard arguments, if agents instead have a rational expectation, the RE solution is

\[ P^{i;RE}_t = \frac{\delta a^{1-\gamma} \rho_c}{1 - \delta a^{1-\gamma} \rho_c} D_t \quad j \in \{A, H\} \]

(11)

where \(\rho_c = E[(e^{\epsilon^c}_{t+1})^{1-\gamma} e^{d^c}_{t+1}] = e^{(1+\gamma)\frac{d^c}{2}} e^{-\gamma \rho_c \delta^c s^d}\). Prices are governed by the dividends and are
equalized.

We now characterize the equilibrium outcomes under learning. Following AMN, we make the technical assumption that the endowment is sufficiently large (see Appendix 9.3). We define subjective expectations of the (risk-adjusted) capital gains $\beta_t^j$ as

$$
\beta_t^j \equiv E_t^P [(\frac{C_{t+1}}{C_t})^{-\gamma} (\frac{P_t^j}{P_t})] \quad j \in \{A, H\}.
$$

(12)

(12) and (10) give rise to the asset pricing equations in equilibrium

$$
P_t^j = \frac{\delta a^{1-\gamma} \rho_t D_t}{1 - \delta \beta_t^j} \quad j \in \{A, H\}.
$$

(13)

We see that price $P_t^j$ is larger when agents expect higher capital gains $\beta_t^j$, and a learning model can generate price differences when $\beta_t^A \neq \beta_t^H$, even if two stocks share the same dividends $D_t$. As $\delta \beta_t^j$ in the denominator is close to 1, a small difference in expected capital gains across the twin shares can manifest as a large difference in prices. Despite knowing that the payoffs of the two shares are identical, agents can have different expected (risk-adjusted) capital gains $\beta_t^j$ across shares as they do not know the equilibrium pricing function. The expected (risk-adjusted) returns $E_t^P [(\frac{C_{t+1}}{C_t})^{-\gamma} \frac{P_t^{j+1}+D_t+1}{P_t}]$ need to be equalized across the twin shares as in equation (10). Given different $\beta_t^j$, the current prices of the A- and H-shares need to differ in a way such that the expected (risk-adjusted) dividend yields $E_t^P [(\frac{C_{t+1}}{C_t})^{-\gamma} \frac{D_t+1}{P_t}]$ achieve this equalization.

Generating different prices hinges on $\beta_t^A \neq \beta_t^H$. This appears to be natural in light of the fact that actual capital gains did in fact differ across these twin shares, even before the Connect program. We now turn to show a system of beliefs about prices that formally justifies this.

### 4.4 Belief-Updating Rule

This section fully specifies the subjective probability distribution for prices in $\mathcal{P}$, and the optimal belief-updating rule for the subjective beliefs $\beta_t^j$. We start with the popular assumption in the adaptive learning/IR literature—namely, that agents believe risk-adjusted capital gains are the sum of a persistent and a transitory component motivated by the existence of periods in which
the price–dividend ratio increases or falls persistently

\[
\left( \frac{C_t}{C_{t-1}} \right)^{-\gamma} \frac{P^j_t}{P^j_{t-1}} = b^j_t + e^j_t, \quad e^j_t \sim \text{i.i.d. } N(0, \sigma^2_{e^j}) \quad j \in \{A, H\} \\
\]

(14)

\[
b^j_t = b^j_{t-1} + \xi^j_t, \quad \xi^j_t \sim \text{i.i.d. } N(0, \sigma^2_{\xi^j})
\]

where \(b^j_t\) are persistent components and \(e^j_t\) are transitory components. In this section we assume all innovations are independent of each other, but in subsection 7.1 we consider a more general case where agents believe that the prices of two shares correlated.

Along with other arguments as found in subsection 4.6, one justification for this belief system is that it is compatible with RE: the setup (14) encompasses the RE beliefs (11) as a special case. Specifically, when agents believe that \(\sigma^2_{\xi^j} = 0\), and \(b^j_0 = a^{1-\gamma}\rho_e\), prices are as given by RE equilibrium prices in all periods. Therefore, this gives a sense in which the beliefs of agents are "close" to RE, as long as we consider small values for \(\sigma^2_{\xi^j}\) and \(\beta^j_0\) close to \(a^{1-\gamma}\rho_e\) (as we will).

In the following we allow for nonzero variances \(\sigma^2_{\xi^j}\)—that is, for the presence of persistent time-varying components \(b^j_t\). Agents observe the realizations of the risk-adjusted capital gains, but not the persistent and transitory components separately. By construction, agents’ expected capital gains \(\beta^j_t\) are their estimation of \(b^j_t\) while using information up to period \(t\). Forecasting the future capital gains by estimating the persistent components \(b^j_t\) engenders a filtering problem. For normally distributed initial priors \(b^j_0 \sim N(\beta^j_0, \sigma^2_{0,j})\), the optimal steady state Kalman filter gives rise to a belief-updating rule:

\[
\beta^j_t = \beta^j_{t-1} + \alpha^j \left[ \left( \frac{C_{t-1}}{C_{t-2}} \right)^{-\gamma} \frac{P^j_{t-1}}{P^j_{t-2}} - \beta^j_{t-1} \right] \quad j \in \{A, H\} \\
\]

(15)

where \(\alpha^j\) are optimal (Kalman) gains that measure the speed at which the agents update their beliefs with new information. It is well known that \(\alpha^j\) increases in the signal–noise ratio \(\frac{\sigma_{\xi^j}}{\sigma_{e^j}}\). We estimate \(\alpha^j\) to be small, as in subsection 4.5; this is compatible with a small \(\sigma^2_{\xi^j}\) implying a small deviation from RE. Iterating (15) backward shows that \(\beta^j_t\) is a geometric weighted average of the history of actual capital gains.

The belief-updating scheme (15) and the pricing equation (13) generate rich price dynamics
that arise from a self-referential channel between the stock price $P_j^t$ and the subjective beliefs $\beta_j^t$. According to equation (13), a high (low) $\beta_j^t$ leads to a high (low) realized stock price $P_j^t$. A high $P_j^t$ reinforces subjective beliefs, and induces an even higher (lower) $\beta_j^{t+1}$ through equation (15), and thus leads to a higher (lower) $P_j^{t+1}$, and so on. The difference between $\beta_t^A$ and $\beta_t^H$ can originate from a small difference in initial beliefs, in the Kalman gains $\alpha^j$, or past realized actual capital gains; the difference can also stem from the addition of expectation shocks (see subsection 6.2, below). Given the self-referential property of the model, a difference in beliefs can produce persistent price differences between A- and H-shares.\(^{37}\)

### 4.5 Quantitative Performance

This subsection evaluates the quantitative performance of the model in which agents learn about prices. The model is simulated at a weekly frequency.\(^{38}\) We set the relative risk-aversion $\gamma = 5$ following AMN, then calibrate the mean and standard deviation of the dividend growth $a$, $\sigma_{\Delta D/D}$, the standard deviation of the consumption growth $\sigma_{\Delta C/C}$, and the correlation between consumption growth and dividend growth $\rho_{c,d}$ using the financial data of SSE and Chinese macroeconomic data. The data sources are described in the Appendix 9.7. In particular, we follow the method as in Campbell (2003) to construct the quarterly dividend. Then, we use quarterly dividend and consumption data to calibrate weekly parameters following Campbell and Cochrane (1999).\(^{39}\) We also calibrate $\delta$ to match the averaged 4% annually deposit interest rate—that is, we derive $\delta$ such that the corresponding weekly interest rate matches the annual deposit interest rate in line with the no-arbitrage condition. We set initial subjective expectations $\beta_0^A$, $\beta_0^H$, and $\beta_1^H$ at their RE values, and calibrate $\beta_1^A$ to be slightly larger than $\beta_1^H$ such that we have an initial 3% premium.

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\(^{37}\)As is standard in this literature agents’ subjective beliefs $\beta_j^t$ are truncated by a projection facility. This is detailed in Appendix 9.4.

\(^{38}\)Week 1 is the week when the Connect program was initiated, and week 0 is one week previous. We use a weekly frequency for two reasons. First, the data sample becomes much smaller if we aggregate it to a monthly or quarterly frequency, since the sample starts only in November 2014. Second, high-frequency financial data are widely used in the literature: for example, Lucca and Moench (2015) used daily and hourly data and found that there is a pre-Federal Open Market Committee announcement excess return.

\(^{39}\)We divide the average quarterly growth rate of dividend (consumption) by 13 to get the weekly mean and obtain the weekly standard deviation of dividend (consumption) growth by dividing its quarterly counterpart by $\sqrt{13}$.

\(^{40}\)We don’t model the deposit explicitly in the model since it is not our concern.
We apply the method of simulated moments (MSM) to estimate $\alpha^A$ and $\alpha^H$, matching the mean, standard deviation, and persistence of the AH premium. Table 1 contains the parameter values. The estimation results show that $\alpha^A > \alpha^H$, which implies agents perceive the signal-noise ratio of A-shares as being larger than that of H-shares i.e., $\frac{\sigma_{\epsilon^A}}{\sigma_{\epsilon^A}} > \frac{\sigma_{\epsilon^H}}{\sigma_{\epsilon^H}}$.\(^{42}\) Intuitively, agents tend to adjust their beliefs about A-shares more quickly if they believe that the variation in the trend component contributes relatively more to the fluctuation of A-share prices than to that of H-share prices.

Table 1: Calibration

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$\gamma$</td>
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<tr>
<td>$\sigma_{\Delta D/D}$</td>
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<tr>
<td>$\sigma_{\Delta C/C}$</td>
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<tr>
<td>$\alpha^H$</td>
<td>0.0014</td>
</tr>
</tbody>
</table>

Figure ?? shows realizations of the time-series outcomes for the A-share prices and H-share prices of the dual-listed companies, generated from simulating the estimated model; we undertake these realizations for the same number of weeks as the number of observations in our data sample. Figure ?? shows the corresponding simulated AH premium. The simulated time series displays persistent price differences similar to those in our data (although it could generate reversed higher H-share prices); in this sense, the model passes an informal "eyeball" test. In addition, we find that the model can explain the Chinese stock market volatility well; see Appendix 9.6, which also lends support to the model.

Table 2 reports the quantitative results. Column 1 contains the targeted moments, and column 2 shows the moments of the actual data from November 2014 to June 2019. Meanwhile, column 3

\(^{41}\)As discussed in section 6.2, higher beliefs for A-shares can come from a positive expectation shock. In the Appendix 9.5, we also use the actual stock prices and the estimated Kalman gains to recover initial beliefs and find similar ones.

\(^{42}\)Suppose that processes (14) are the data–generating process (in fact, as is in subsection 4.6, (14) are compatible with actual data), The maximum likelihood estimation of (14) from using the actual data also supports this inequality.
reports the 95% intervals of the model’s simulated moments. We find that the mean and standard deviation of the data are located within the intervals, although the model generates a slightly more persistent AH premium than do the data.\textsuperscript{43} Therefore, the learning model is broadly consistent with the data, in the sense that it can produce data-like behavior.

Figure 3 graphically displays the fact that while A- and H-share prices usually move in the same direction, those of A-shares often adjust more quickly than H-share prices. The formal statistics capturing these characteristics are the correlation \( \text{corr}(P^A_t, P^H_t) \), \( \text{corr}(P^A_t - P^H_t, P^H_t) \) and standard deviation ratio \( \sigma(P^A_t)/\sigma(P^H_t) \). Column 2 of Table 3 reports the empirical statistics. We find a high positive correlation \( \text{corr}(P^A_t, P^H_t) \). Additionally, the positive \( \text{corr}(P^A_t - P^H_t, P^H_t) \) indicates that A-share prices tend to increase (decrease) more as H-share prices increase (decrease), while \( \sigma(P^A_t)/\sigma(P^H_t) > 1 \) implies that A-share prices are more volatile. The learning model results reported in column 3 of Table 3 quantitatively replicate these moments, as data moments are within 95% intervals of the simulated moments— even though they are not targeted when estimating the model.

The learning model generates targeted and nontargeted moments similar to what is seen in the data, even as the RE and Bayesian RE models discussed in section 3 have difficulty producing the data-like AH premium. These quantitative results confirm the key role of learning about prices in equity pricing.

\begin{table}[h]
\centering
\caption{Simulated Targeted Moments} \label{tab:sim-targeted}
\begin{tabular}{l|c|c}
\hline
Targeted Moments & Data & Model (95\% interval) \\
\hline
\( E(\frac{P^A_t}{P^H_t} \times 100) \) & 126.68 & [93.54128.74] \\
\( \sigma(\frac{P^A_t}{P^H_t} \times 100) \) & 6.98 & [3.7926.01] \\
\( \rho(\frac{P^A_t}{P^H_t}) \) & 0.88 & [0.960.99] \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\caption{Simulated Non-targeted Moments} \label{tab:sim-nontargeted}
\begin{tabular}{l|c|c}
\hline
Non-targeted Moments & Data & Model (95\% interval) \\
\hline
\( \text{corr}(P^A_t, P^H_t) \) & 0.91 & [0.670.99] \\
\( \text{corr}(P^A_t - P^H_t, P^H_t) \) & 0.18 & [-0.140.94] \\
\( \sigma(P^A_t)/\sigma(P^H_t) \) & 1.20 & [1.072.37] \\
\hline
\end{tabular}
\end{table}

\textsuperscript{43}By introducing the preference shock to \( \delta \), the persistence could be reduced.
4.6 Testing for the Rationality of Price Expectations

The idea of IR is that even though agents do not know the equilibrium pricing function, they entertain beliefs that are not large deviations from rational expectations, in the sense that agents do not make large, easily detected mistakes. As explained in subsection 4.4, choosing small $\sigma^2_{\xi^j}$ and initial conditions $\beta_0^j$ close to $a^{1-\gamma} \rho_t$ gives a precise sense of "closeness" of beliefs to RE beliefs.

In addition, agents can test their belief system against the actual data and reject their subjective model if it leads to big mistakes. In this section, we examine whether agents would be able to reject their beliefs if they were to test them.

For this purpose, we use a set of testable restrictions implied by the agents' belief system (4) (5) (15) developed by AMN to test the belief system against actual data.\textsuperscript{44} Denote $x_t \equiv (e_t, D_t/D_{t-1}, C_t/C_{t-1})$, where $e_t \equiv \Delta(C_{t-1}^t)\gamma P_{t-1}^{D_t}$, with $\Delta$ representing the first difference operator. These restrictions are listed as follows:

Restriction 1: $E(x_{t-i}e_t) = 0$ for all $i \geq 2$,

Restriction 2: $E((D_t/D_{t-2} + D_{t-1} + C_t/C_{t-1} + C_{t-1} + C_{t-2})e_t) = 0$,

Restriction 3: $b'_DC \sum_{DC} b_{DC} + E(e_t e_{t-1}) < 0$,

Restriction 4: $E(e_t) = 0$,

where $\sum_{DC} \equiv var\left(\frac{D_t}{D_{t-1}}, \frac{C_t}{C_{t-1}}\right)$ and $b_{DC} \equiv \sum_{DC}^{-1} E\left((\frac{D_t}{D_{t-1}}, \frac{C_t}{C_{t-1}})'e_t\right)$.

These four restrictions are necessary and sufficient conditions for the agents' beliefs to be compatible with $\{x_t\}$ in terms of second-order moments.\textsuperscript{45} Under standard assumptions, any process that satisfies these testable restrictions can be generated by the postulated belief system in terms of its autocovariance function.

Table 4 reports the test statistics associated with Restrictions 1–4.\textsuperscript{46} Column 2 contains the test statistic values, while the final column reports the statistics' 5% critical values. This table shows that the test statistics are, in all cases, below their critical values, and often by a wide margin. Agents would find the observed stock prices compatible with their belief system, as they would not reject their subjective model if they were to test it against actual data. We conclude

\textsuperscript{44}AMN show that the postulated belief system is compatible with the simulated data.

\textsuperscript{45}See AMN for the proof and details.

\textsuperscript{46}Given the availability of consumption and dividend data, here we use monthly data from January 2006.
that the belief system is reasonable, given the behavior of the data.

Table 4: Testing Subjective Beliefs against Actual Data

<table>
<thead>
<tr>
<th>Restriction 1 using $D_t^i$</th>
<th>Test Statistics for A (H)</th>
<th>5% Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{D_t^i}{C_t^i}$</td>
<td>2.81 (0.76)</td>
<td>9.48</td>
</tr>
<tr>
<td>Restriction 1 using $C_t^i$</td>
<td>4.02 (4.77)</td>
<td>9.48</td>
</tr>
<tr>
<td>Restriction 1 using $\Delta(C_t^i)$</td>
<td>2.13 (2.55)</td>
<td>9.48</td>
</tr>
<tr>
<td>Restriction 2</td>
<td>0.04 (0.15)</td>
<td>5.99</td>
</tr>
<tr>
<td>Restriction 3</td>
<td>-3.55 (-3.60)</td>
<td>1.64</td>
</tr>
<tr>
<td>Restriction 4</td>
<td>0.002 (0.001)</td>
<td>3.84</td>
</tr>
</tbody>
</table>

5 Convergence Trading

Lamont and Thaler (2003) consider different prices between the twin shares a violation of the law of one price. With regard to this issue, they state that “First, some agents have to believe falsely that there are real differences between two identical goods, and second, there have to be some impediments to prevent rational arbitrageurs from restoring the equality of prices that rationality predicts” (p.192). In section 4, we answer the first question of how investors could believe that A- and H-shares with identical payoffs are different. Now, in the context of agents learning about prices, we switch to what prevents arbitrageurs from bringing stock prices to their fundamental values.

Arbitrageurs with superior information expect the prices of two assets with the same or similar fundamentals to converge in the future. In fact, when agents learn prices, the prices in equilibrium will also converge in the long run. According to the results cited in chapter 7 of Evans and Honkapohsa (2001), learning agents’ price expectations $\beta_t^{A(H)}$ (as seen in section 4) should converge in distribution as

$$
\lim_{t \to \infty} \beta_t^j \sim N(\beta^{RE}, \alpha^j Z^j), j = A, H
$$

where $\alpha^j$ and $Z^j$ are numbers given in their Theorem 7.9. Hence, the difference in price expectations (i.e. $\beta_t^A - \beta_t^H$) converges in distribution with a mean of zero. Given that the AH premium is caused by different price expectations, the initialization of the Shanghai–Hong Kong Stock Connect program can promote price convergence in the long run. However, the learning agents do not
know that $\beta_t^{A(H)}$ converges and already take the optimal strategy compatible with their beliefs. Hence, they will not act as arbitrageurs.

Arbitrageurs typically implement the convergence trading strategy that takes long positions in an asset believed to be undervalued, and short positions in an asset believed to be overvalued. A well-known example is the convergence trading of the hedge fund Long-Term Capital Management (LTCM), which bet on the convergence of prices of Royal Dutch and Shell.\footnote{Edwards (1999) and Lamont and Thaler (2003) document that LTCM expected the convergence of bond yields in emerging market countries and the US. Hence, they bought bonds from emerging markets and short-sold US government bonds in 1997. At the same time, LTCM also bet that share prices of the dual-listed company Royal Dutch and Shell would converge because of the same fundamentals. However, the spread of bond yields and price differences of stocks widened unexpectedly, which made LTCM incur a large loss from this strategy and led to the near-collapse of LTCM.} However, the convergence trading is not always profitable, especially in a “short-term world”.\footnote{See the section 7 in Lamont and Thaler (2003)} LTCM incurred a large loss from taking this strategy. As mentioned in the introduction, the news media reported that some arbitrageurs in the AH-market also lost heavily and quickly abandoned convergence trading; this is due to the limits of arbitrage, as follows.

Among others, we list several prominent limits. First, Shleifer and Vishny (1997) argue that arbitrage is conducted by a relatively small number of professional and highly specialized investors. Second, as Edwards (1999) and Shleifer and Vishny (1997), among others, have pointed out, naked short-selling (i.e., long-buying) is not allowed in the US market. Regulation T requires that arbitrageurs looking to short-sell (long-buy) a security must have an initial margin requirement of 50% of the market value of the security in their credit accounts; effectively, this means that it requires short (long) trades to have 150% of the value of position at the time the position is created. The same situation holds in China.\footnote{The detailed regulations of short selling can be found in the official website of China Securities Regulatory Commission. (http://www.csrc.gov.cn/pub/xizang/xxfw/tzzsyyd/201003/t20100324_178727.htm)} Third, arbitrageurs must also comply over time with the maintenance ratio regulation, which states that the ratio of the market value of assets (the initial margin plus cash from short-selling and the asset value from long-buying) to the market value of debt in the credit accounts must be at least 125%.\footnote{Note that this maintenance ratio has to be at least 150% at the time the position is created as a result of the initial margin requirement.} In the case of China, the minimum maintenance ratio is 130%. Whenever that ratio is below the threshold value, arbitrageurs receive...
margin calls and are forced to remargin, to provide additional collateral. Involuntary liquidation can occur if there is not enough capital to remargin. Fourth, Shleifer and Vishny (1997) propose that arbitrageurs are very likely to encounter the liquidation issue, as funding external to them is usually performance-based. Fifth, in the mainland market, arbitrageurs have an additional restriction of a one-year maximal duration of margin trading, whereas there is no such regulation in US markets. Sixth, it is very difficult—and may even be impossible—for arbitragers to design a strategy by which to short-sell a stock index without corresponding derivatives.

Moreover, we find that internally rational agents’ learning about prices also serves as an impediment to arbitrage since the price differences induced by learning may not converge, and could even diverge in the short run; this might incur arbitrageur losses and a high odds of margin calls in the short run. To illustrate this, we study the risk of convergence trading by undertaking a simple simulation experiment as follows. We assume only a small number of arbitrageurs in line with Shleifer and Vishny (1997), who basically take the prices as given and start to take arbitrage in period $T$. Given that the A-share prices are higher than those of the H-shares in period $T$, arbitrageurs wish to short-sell A-shares and long-buy H-shares. We assume arbitragers have enough cash in hand as the initial margin. To short-sell one unit of A-shares, they deposit cash of $0.50 \times P_T^A$ as the initial margin required, borrow one unit of A-shares from brokers, and then keep in the credit account as collateral the cash of $P_T^A$ from the short position. Meanwhile, we assume arbitrageurs borrow $P_T^A$ amount of money with the same amount of initial margin of $0.50 \times P_T^A$ to buy $P_T^A/P_T^H$ units of H-shares as the long position. (Here we ignore the broker’s charged fee, interest rate, other transaction costs, and dividend payments, as they are relatively small.) In this case, the assets in the arbitrageur’ account includes the total initial margin of $P_T^A$, cash received from the short position $P_T^A$, and the market value of the long position $P_T^A/P_T^H \times P_T^H$. Hence, the maintenance ratio $mr$ in period $T + t$, the ratio of the market value of assets to that of debt, is

$$mr_{T+t} = \frac{2P_T^A + P_T^A/P_T^H \times P_T^H}{P_T^A/P_T^H + P_T^A}$$

We do not model arbitrageur’ decision–making about remargining except to point out later that the odds of margin calls are high. In this case, the profit $\pi_t$ by terminating the convergence trading
in period $T + t$ is

$$\pi_{T+t} \equiv (P^H_{T+t} * P^A_T / P^H_T - P^A_T) + (P^A_T - P^A_{T+t})$$

$$= P^H_{T+t} * P^A_T / P^H_T - P^A_{T+t}$$

where $P^H_{T+t} * P^A_T / P^H_T - P^A_T$ is profit from long position, and $P^A_T - P^A_{T+t}$ is profit from short position.

Consider a scenario where the arbitrageurs observing the AH premium start to implement their margin trading in period $T = 100$.\(^{51}\) We Monte-Carlo simulate the model with each path covering 52 periods (1 year) from period $T$. The actual A- and H-share prices are substituted into the belief updating scheme (15) to recover the learning agents’ capital gain expectations in period $T$ as the initial beliefs for simulation.\(^{52}\)

Our simulation results shows that convergence trading probably experiences marginal calls, as A- and H-share prices can diverge in the short run as a result of learning about prices. The probability of receiving a margin call in each period is obtained by calculating the fraction of paths with $mr_{T+t} < 130\%$ among all the simulated paths over each period. Although we assume arbitrageurs already have large amounts of cash to satisfy the initial margin requirement, the maximal probability of margin calls over all periods still reaches as high as 9.6%.

Furthermore, we explore whether arbitrageurs will make positive profits through convergence trading. Table 5 displays the means, standard deviations of $\pi_{T+t}$, and probabilities of negative profits \(i.e.\) $\pi_{T+t} < 0$ for $t = 13, 26, 39$ and 52, corresponding to three months, six months, nine months, and one year after the creation of the positions, respectively. As one can see in Table 5, the means of $\pi_{T+t}$ are negative, and the standard deviations are large, implying that convergence trading is risky; the probabilities of negative profits are high. Figure ?? plots the distribution of profits for $t = 52$, the left-fat-tail of which indicates a high probability of substantial loss.\(^{53}\) This left-tail distribution and the high odds of margin calls induced by agents learning about prices work together to prevent a small number of arbitrageurs from undertaking risky convergence trading.

\(^{51}\)Note that week 1 is the week in which the Connect program was initiated.

\(^{52}\)We keep the consumption growth at its mean value because weekly consumption data doesn’t exist.

\(^{53}\)If we take maintenance ratio requirements into account, convergence trading would be riskier. This table can be regarded as giving the lower bound for the loss.
Our results align with those in a considerable body of literature on arbitrage, including the work of Shleifer and Vishny (1997), Xiong (2001), and De Jong, Rosenthal, and Van Dijk (2009). Although arbitrage in the long run is profitable\textsuperscript{54}, the limits of arbitrage along with the short-run price divergence cause arbitrageurs to fail in “a short-term world”.

\begin{table}[h]
\centering
\caption{The Statistics of Profits}
\begin{tabular}{llll}
\hline
 & Mean & Std & Pr ($\pi_{T+t}<0$) \\
3m & -0.798 & 16.050 & 0.425 \\
6m & -3.223 & 29.584 & 0.412 \\
9m & -4.432 & 37.645 & 0.388 \\
1y & -4.910 & 41.626 & 0.382 \\
\hline
\end{tabular}
\end{table}

6 Extensions

In this section, we discuss some extensions to the benchmark learning model in section 4. We exhibit cross-learning scheme in section 6.1 and expectation shocks in section 6.2.

6.1 Cross-Learning Scheme

Figure ?? displays that A- and H-share prices are highly positively correlated. By observing this high correlation, agents probably think that the prices of one share can provide information for the other share. Therefore, instead of assuming independent price processes (14), it is natural to model agents’ subjective belief about prices as the cross-learning scheme

\begin{equation}
\begin{bmatrix}
\begin{bmatrix}
\frac{C_t}{C_{t-1}} - \frac{P_{A_t}}{P_{t-1}} \\
\frac{C_t}{C_{t-1}} - \frac{P_{H_t}}{P_{t-1}}
\end{bmatrix} \\
\begin{bmatrix}
\frac{b_t^A} \\
\frac{b_t^H}
\end{bmatrix}
\end{bmatrix} = \begin{bmatrix}
\frac{b_{t-1}^A} \\
\frac{b_{t-1}^H}
\end{bmatrix} + \begin{bmatrix}
\frac{\epsilon_t^A} \\
\frac{\epsilon_t^H}
\end{bmatrix}
\end{equation}

where $\frac{b_t^A}{b_t^H}$ are jointly-normal and $\frac{\epsilon_t^A}{\epsilon_t^H}$ are jointly-normal, and $\xi_t^A, \xi_t^H$ are jointly-normal and $\xi_t^A, \xi_t^H$ are jointly-normal. The independent subjective belief about A-\textsuperscript{54}Because prices converge in the long-run.
and H-share prices is a special case of the cross-learning scheme when $R$ and $Q$ are reduced to be diagonal matrices.

Agents optimally update their beliefs according to:

$$
\begin{bmatrix}
\beta^A_t \\
\beta^H_t
\end{bmatrix} =
\begin{bmatrix}
\beta^A_t \\
\beta^H_t
\end{bmatrix} +
\begin{bmatrix}
\alpha^A & \alpha^C \\
\alpha^C & \alpha^H
\end{bmatrix}
\begin{bmatrix}
(C_{t-1}/C_{t-2}) - \gamma P^A_{t-1}/P^A_{t-2} - \beta^A_{t-1} \\
(C_{t-1}/C_{t-2}) - \gamma P^H_{t-1}/P^H_{t-2} - \beta^H_{t-1}
\end{bmatrix}.
$$

They use not only the newly observed A-share prices (H-share prices) but also the H-share prices (A-share prices) to update their beliefs about future capital gains of A-shares (H-shares).\footnote{Define $X$ such that $X(X + R)^{-1}X = Q$. The Kalman gain vector $K = \begin{bmatrix} \alpha^A \\ \alpha^C \end{bmatrix} = X(X + R)^{-1}$.}

The pricing equation is still (15). We use MSM to estimate the parameters $[\alpha^A \alpha^H \alpha^C]$ to match the targeted moments in section 4 as well as the correlation of A- and H-share prices. $\alpha^A$ and $\alpha^H$ are estimated to be 0.0033 and 0.0015, respectively, similar to the estimation found in section 4, while $\alpha^C = 0.0002$ is smaller than them with one order of magnitude. Meanwhile, we find that the 95% intervals of model moments in Table 6 are very close to those in section 4.

<table>
<thead>
<tr>
<th>Moments Data Model (95% interval)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(P^A_t / P^H_t * 100)$</td>
</tr>
<tr>
<td>$\sigma(P^A_t / P^H_t * 100)$</td>
</tr>
<tr>
<td>$\rho(P^A_t / P^H_t * 100)$</td>
</tr>
<tr>
<td>$corr(P^A_t, P^H_t)$</td>
</tr>
</tbody>
</table>

### 6.2 Learning with Expectation Shocks

Expectation shocks are shown to be important for business cycle and asset prices, (e.g., Bullard, Evans and Honkapohja 2008 and Milani 2011, 2017). Expectation shocks can be understood as waves of optimism and pessimism on top of the the expected future capital gains from the constant gain learning from the baseline model.

We incorporate expectation shocks into the learning model by modifying the belief updating
scheme as follows

\[
\beta^j_t = \beta^j_{t-1} + \alpha^j \left( (C^j_{t-1})^{-\gamma} \frac{P^{j}_{t-1}}{P^{j}_{t-2}} - \beta^j_{t-1} \right) + \epsilon^j_t \quad j \in \{A, H\}
\]  

(17)

where \(\epsilon^j_t \sim i.i.d. N(0, (\sigma^j)^2)\) are the expectation shocks.\(^{56}\) The different realized expectation shocks give rise to the difference in \(\beta^A_t\) and \(\beta^H_t\). \textit{Ceteris paribus}, higher expectation shocks for A-shares \(\epsilon^A_t > \epsilon^H_t\) leads to larger expected capital gains for A-shares \(\beta^A_t > \beta^H_t\).

Many factors can induce waves of optimism and pessimism about future capital gains when investors are internally rational, including more-liquid markets, new market participants, or even some special events in China around those dates. In particular, Huang, Miao, and Wang (2019) discuss the media’s widespread and optimistic reports and the general optimistic narrative of a reform-based bull market for A-shares in 2014 and early 2015.

Another well-known factor is the ups and downs in the broad stock market indexes of the markets where companies dual-list. Indeed, we find that return differentials in the A-shares versus H-shares of dual-listed companies comove with the relative stock market indexes, as in Froot and Dabora (1999). Following that study, we show this comovement feature by running a regression of the log return differentials in the local currency \(r_{A-H}\) on log returns in the Shanghai market index \((SCI)\) and the Hong Kong market index \((HSI)\) as well as the log change in exchange rate \(ex\) as

\[
r_{A-H,t} = \alpha + \beta SCI_t + \delta HSI_t + \gamma ex_t + \varepsilon_t
\]

Two-stage least square is used to avoid the potential endogeneity issue. We use Shanghai Small/Mid Cap Index (specification 1) and Shenzhen Component Index (specification 2) as the instrumental variables for \(SCI\), and Hang Seng Medium Cap Index (HSMC) as the instrumental variable for \(HSI\). The frequency is 5 trading days, and the sample is from Nov 2014 to June 2019. Table 7 exhibits the comovement, with the coefficients on the market indexes \(SCI\) and \(HSI\) being significant at the 1% confidence level. Froot and Dabora (1999) conjecture that the

\(^{56}\)Adam, Marcet and Beutel (2017) provide a Bayesian foundation for this updating rule with expectation shocks.
comovement could be induced by market-specific sentiment shocks that we think are compatible with IR learning and, in this study, are modeled as expectation shocks.\footnote{When investors know the equilibrium pricing functions—as in RE or Bayesian RE—prices are determined by expectations about future fundamentals. Hence, comovement should not appear in these models.}

<table>
<thead>
<tr>
<th>Table 7: Froot Dabora Regression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specification 1</td>
</tr>
<tr>
<td>SCI</td>
</tr>
<tr>
<td>HSI</td>
</tr>
<tr>
<td>(e_x)</td>
</tr>
<tr>
<td>(R^2)</td>
</tr>
</tbody>
</table>

We also simulate the model to understand the effects of expectation shocks on the AH premium and convergence trading. Instead of estimating the parameter values of expectation shocks as in Milani (2011), we set the standard deviations \(\sigma^j\) such that the standard deviations of \(\beta^j\) are 10% larger than the \(\beta^j\) in section 4. Table 8 contains the model moments. We find that while expectation shocks lead to higher and more volatile premiums, the model moments are still broadly compatible with the data. The probability of receiving marginal calls \((mr_t < 130\%)\) becomes larger and can reach as high as 14.4% as A- and H-share prices are more likely to diverge in the short-run. Furthermore, as one can see in Table 9, we see a more negative mean and larger standard deviation of convergence trading profit \(\pi_{T+t}\), as well as a higher probability of negative profit, as seen in the more-skewed left-fat-tailed distribution of profits in Figure ??.

<table>
<thead>
<tr>
<th>Table 8: Model Simulated Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moments</td>
</tr>
<tr>
<td>(E(\frac{P_A}{P_H} * 100))</td>
</tr>
<tr>
<td>(\sigma(\frac{P_A}{P_H} * 100))</td>
</tr>
<tr>
<td>(\rho(\frac{P_A}{P_H} * 100))</td>
</tr>
</tbody>
</table>
### Table 9: The Statistics of Profits

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std</th>
<th>Pr ($\pi_{T+t}&lt;0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3m</td>
<td>-2.455</td>
<td>20.581</td>
<td>0.472</td>
</tr>
<tr>
<td>6m</td>
<td>-7.891</td>
<td>39.116</td>
<td>0.469</td>
</tr>
<tr>
<td>9m</td>
<td>-10.938</td>
<td>47.600</td>
<td>0.465</td>
</tr>
<tr>
<td>1y</td>
<td>-13.356</td>
<td>54.247</td>
<td>0.468</td>
</tr>
</tbody>
</table>

#### 7 Discussion

In section 3, we consider RE and Bayesian RE models with heterogeneous agents who have pricing knowledge. Obviously, the analysis in section 3 does not cover all possible theories where the knowledge of the equilibrium pricing function is preserved. In this section, we discuss informally other possible explanations for the AH premium. While future research may show that these can explain some of the differences in stock prices, it seems unlikely that they can compensate for the large and persistent premium we see.

Exchange rate appears to play a role in explaining the AH premium, because A-shares are denominated in RMB while H-shares denominated in HKD. We conjecture that a no-arbitrage condition in the stock market, similar to the interest rate parity, would hold—that is, the expected return differential between A-shares (in RMB) and H-shares (in HKD) should equal the expected change in the exchange rate. However, the HKD was expected to appreciate against the RMB by an average of 0.02%, as measured by the one-week exchange rate forward from November 2014 to June 2019.\(^{58}\) According to the no-arbitrage condition, the expected appreciation of the HKD should have implied higher average H-share prices. Besides, the HKD is pegged to the USD, and mainland China implements a managed floating exchange rate regime. The standard deviation of the daily exchange rate of the RMB against the HKD for the period from November 2014 to June 2019 was 0.033. In comparison, the exchange rate of the USD against the euro during the same period was 0.048. Hence, RMB–HKD exchange rate risk is relative low.

We have not formally modeled twin shares with different liquidity. It is widely believed that the higher an asset’s liquidity is, the higher its price will be. One popular measure of liquidity

\(^{58}\)The forward rate arguably reflects investors’ expectations about future exchange rate. The expected change is calculated by dividing the forward rate by the spot exchange rate and taking the average over the sample period. Hong Kong dollar was expected to appreciate, on average, by 0.15%, 0.50%, and 0.72% measured by 1-month, 3-month, and 1-year forward rates.
for a stock (index) is the proportion of price-change days over a certain period (Mei, Scheinkman and Xiong 2009). Based on daily data from the 2006–2019 period, the proportion of trading days with price changes for A-shares is 98.69%, while this proportion for H-shares is 97.28%. These numbers suggest that both A- and H-shares are quite liquid, with A-shares just marginally more liquid. In any case, there is no well-accepted model of liquidity in asset markets, and exploring this issue formally is a massive undertaking that extends beyond the scope of this study.

Moreover, transaction costs levied on the trading value in the SSE and HKSE are small; these including the stamping duty, the security management fee, the transfer fee, and the handling fee. The total transaction cost in the HKSE is about 0.118%, according to regulations; that in the SSE is about 0.169%.\(^{59}\) This tiny difference throws doubt on the ability of transaction costs to produce the data-like premium. Additionally, the HKSE’s lower transaction costs might imply slightly higher H-share prices.

There has been a recent interest in rational bubbles in stock markets (e.g., Martin and Ventura 2012; Miao and Wang 2018). In the literature relevant to the current study, rational bubbles facilitate liquidity and relax borrowing constraints. The work of Miao and Wang pertains to infinitely lived agent models, and they found that when agents borrow against the market value of stocks, there is the potential for quantitatively relevant bubbles to emerge. To the best of our knowledge, however, no study has addressed rational bubbles in two assets that share the same fundamentals in infinitely lived agent models. In principle, there could be equilibria where there is a rational bubble in one stock but not in the other; this situation would justify the AH premium. However, since both A- and H-shares can be used as collateral through share-pledging to secure loans\(^{60}\), achieving such equilibria appears to be challenging. In addition, we conjecture that it is likely that if such an equilibrium were found, it would display an explosive AH premium, unlike the stable one found in the data. In any case, a more formal study into this matter would be of interest.

\(^{59}\)See the sources of transaction costs in Appendix 9.7.
\(^{60}\)See Li et al. (2019) for A-share pledging and the link below for H-share pledging. (https://en-rules.hkex.com.hk/node/2505)
For example, the SSE has daily price limits of 10% on regular stocks and 5% on special treatment stocks, while the HKSE has no such limits. Second, the trading system in SSE is "T+1"; in HKSE, it is "T+0". Third, these stock exchanges differ in terms of their regular trading sessions, premarket opening sessions, and block trade. All of these microstructure differences might affect stock prices. Indeed, Chen et al. (2019) found that the daily price limit scheme amplifies rather than stabilizes market volatility, given the destructive strategies of large speculators. The question is how these microstructure differences would generate a large positive AH premium.

The current study presents a natural and parsimonious IR learning theory by which to quantitatively rationalize the AH premium, and it emphasizes the critical role of agents’ price expectations. Nevertheless, we cannot rule out other potential explanations, including liquidity, a rational bubble, and the market microstructure, as discussed above. Formal examinations of these factors are left to future research.

8 Conclusion

This study explains the AH premium. Since the AH-market is a large one, understanding it is essential for market investors; since that market also provides a natural experiment by which to test expectation formation in asset-pricing models. We show that many versions of rational expectations (RE) and Bayesian RE asset equilibria where agents know the equilibrium pricing function fail to generate a data-like AH premium; these include models with agent heterogeneities and dividend taxes. We also put forward some other possible explanations.

Our model of learning about prices explains the price differences in dual-listed companies in a very natural way, as small differences in beliefs about the stocks in different markets provide a "nearly" self-fulfilling prophecy of the prices being different. When internally rational agents learn about the stock prices, the A- and H-share prices will differ in equilibrium simply because

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62 T+1 only allows investors to get ownership and sell the shares the next day after they buy the shares. By contrast, T+0 trading system allows investors to buy and sell shares on the same day.
63 The pre-market is the period of trading activity that occurs before the regular trading session. A block trade is a permissible, noncompetitive, privately negotiated transaction either at or exceeding an exchange determined minimum threshold quantity of shares.
investors expect them to differ. Therefore, we think of the AH premium as a natural experiment that supports learning about prices in terms of stock-price expectation formation. Of course, some factors uncovered by RE and Bayesian RE models could explain part of the story, but they do not seem to be the main characters in this story. Transaction costs, liquidity, and rational bubbles may act as good "supporting actors" in future research, but the main actor appears to be learning about prices.

Although agents do not know precisely the equilibrium pricing function, they hold beliefs that are close to the actual behavior of the data. Agents make "mistakes" that would be statistically undetectable and, therefore, are not really "mistakes." We also show that convergence trading is very much likely to suffer a significant loss, because prices might diverge in the short run. Hence, even if arbitrageurs have superior information, short-run price divergences prevent them from bringing stock prices to their fundamental values.

As the literature points out—and as we discuss in section 5—the limits of arbitrage are strict, and only a small number of professional investors undertake arbitrage. However, as the classical literature shows (e.g., Sharpe 1964; Fama 1965), if the number of arbitrageurs is large, their collective actions should force stock prices to converge to their fundamental values. While in section 5 we assume a zero measure of arbitrageurs as a shortcut, it would be interesting to investigate whether a sizeable positive measure of them with RE trading and with learning agents would ride the wave or lean against the bubble. The conclusion might not be as obvious as it seems, as many studies (e.g., Blume and Easley 2006; Cogley and Sargent 2009; Dumas, Kurshev, and Uppal 2009) point out that it could be that agents with more information would be driven out of the market (in an incomplete market), or that it takes a very long time for learning agents to be driven out of the market.

We have not found appropriate survey data relating to the Chinese financial market—data that could be used to refine the learning story that we propose. Such survey data would also help us understand expectation formation in the Chinese market, and as such should be closely monitored by policy-makers and market participants. If such data are not yet available, it seems a promising avenue to conduct surveys and elicit investor expectations; such work would be in line with that
of Livingston Survey and Shiller Survey for stock price forecasts.

References


9  **Internet Appendix**

9.1  **AB Shares**

A special feature in the mainland Chinese stock market is that several dozen companies have issued twin shares listed on the same exchanges. Twin shares dubbed A- and B-shares have existed since 1993. They have identical dividend and voting rights, but they were traded by different investors, so it is not surprising that they had different prices. A-shares traded with RMB used to be restricted to mainland investors before November 2014. In addition, many companies issued B-shares traded with US dollars were strictly confined to international investors. Mainland investors were allowed to trade B-shares using US dollars since February 2001, but, as discussed in Mei, Scheinkman and Xiong (2009), the difficulties for Chinese citizens to acquire US dollars still serve as severe restrictions for mainland investors to participate in the B-share market. Therefore, A- and
B-share markets were strictly segmented before February 2001 and are still segmented to a large extent for mainland investors. The literature documented that A-shares traded at higher prices than B-shares despite their identical rights, e.g., Fernald and Rogers (2002) and Mei, Scheinkman and Xiong (2009). This premium has been explained in the light of models of segmented markets and heterogeneous agents.

9.2 Algorithms and Simulations

9.2.1 Rational Expectations

We assume the dividend payment takes value of $D^h$ (high) with probability $\text{prob}(D^h) = (1 - \pi)$ and $D^l$ (low) with probability $\text{prob}(D^l) = \pi$ in each period. The dividend payment is assumed to be high $D^h$ in the first period. The two types of agents with rational expectation share the same subjective discount factors i.e. $\delta^1 = \delta^2$ and the endowment is constant i.e. $Y_t = Y$. In this case, the policy function is time-invariant and only depends on the current realization of dividend payment.

Step 1: Draw $N$ series of $M$ periods each, of dividends $\{D^n_t\}_{t=0}^M$ using a $i.i.d$ random number generator with $D_{n,0} = D^h$ for all $n$. Guess a value for $\lambda$ representing the constant ratio of marginal utility in the complete market i.e. $\lambda = \frac{u'(C_t^{1,n})}{u'(C_t^{2,n})}$ and then simulate for consumption \$\{C_t^{1,n}, C_t^{2,n}\}_{t=0}^M\$ using commodity goods market clearing condition given the guess of $\lambda$. Solve for $\lambda$ by iteration to satisfy the following intertemporal budget constraint of type 1 agent:

$$\frac{1}{N} \sum_{n=1}^{N} \sum_{t=0}^{M} \delta^t u'(C_t^{1,n}) (C_{t+j}^{1,n} - D_{t+j}^n - Y) = B_{t-1}^1$$

where $B_{t-1}^1$ is the initial inherited holding of the state-contingent bond. Then we obtain the state-contingent bond price once we get $c(D^l)$ and $c(D^h)$.

Step 2: Let $Z \equiv u'(c(D^l))(c(D^l) - D^l - Y^1)\pi + u'(c(D^h))(c(D^h) - D^h - Y^1)(1 - \pi)$. We obtain $B_{t-1}^1(D^l)$ by
\[ B_{t-1}^1(D^l) = E_t \sum_{j=0}^{\infty} \delta^j \frac{u'(C_{t+j}^{1,n})}{u'(C_{t}^{1,n})} (C_{t+j}^{1,n} - D_{t+j}^n - Y) \]

\[ = (C^1(D = D^l) - D^l) + \frac{\delta}{u'(C^1(D = D^l))(1 - \delta)} Z \forall t. \]

and \( B_{t-1}^1(D^h) = B_{t-1}^1 \) because of the time-invariant policy functions and the assumption of \( D_0 = D^h \). The bond positions of type 2 agents are the opposite to those of type 1 agents as a result of the bond market clearing condition. Similarly, we obtain the stock price.

In our simulation, we let \( \delta = 0.99, \gamma^2 = 1, Y = 0, D^h = 1, D^l = 0.5 \) and \( \pi = 0.5 \). We alter \( \gamma^1 \) while fixing \( \gamma^2 \) to illustrate how the degree of risk aversion affects consumption and bond allocations.\(^6\) When \( \gamma^1 = \gamma^2 \), the two types of agents share the same consumption and don’t hold the state-contingent bonds. As type 1 agents become more risk averse, they want a more smooth consumption profile by holding more \( B^1(D^l) \) seeking to be compensated in the bad states. While varying \( \gamma^1 \) does affect the price of equities, it is consistent with the equalization of \( P^A \) and \( P^H \).

These results are shown in Table 10, where \( x(D^h) \) means the value of \( x \) for \( D_t = D^h \).

| Table 10: Consumption and State-Contingent-Bond Holdings |
|-----------------|-----------------|-----------------|-----------------|-----------------|
|                 | \( \gamma^1 = 1 \) | \( \gamma^1 = 2 \) | \( \gamma^1 = 3 \) | \( \gamma^1 = 4 \) |
| \( C^1(D^h) \)  | 1                | 0.8736          | 0.8045          | 0.7629          |
| \( C^1(D^l) \)  | 0.5              | 0.5514          | 0.5717          | 0.5818          |
| \( B^1(D^l) \)  | 0                | 0.1017          | 0.1416          | 0.1619          |
| \( P^A(D^h) \)  | 100              | 112.64          | 119.55          | 123.71          |
| \( P^H(D^h) \)  | 100              | 112.64          | 119.55          | 123.71          |
| \( P^A(D^l) \)  | 50               | 44.97           | 42.99           | 41.99           |
| \( P^H(D^l) \)  | 50               | 44.97           | 42.99           | 41.99           |

When agents have different subjective discount factors, say \( \delta^1 < \delta^2 \), then \( \frac{u'(c_{t+1}^1)}{u'(c_{t+1}^2)} = \frac{\delta^2}{\delta^1} \frac{u'(c_{t})}{u'(c_{t}^2)}. \)

In this case, the marginal rate of substitution \( MRS_{12} \) is not constant and increases over time.

Impatient type 1 agents consume less and less over time. In the limit, \( \frac{u'(c_{M}^1)}{u'(c_{M}^2)} \to \infty \) as \( M \to \infty \), this means that in the limit type 2 agents consume all the dividends \( c_{M}^2 \to y_M \) while type 1 agents

\(^6\)With state-contingent bonds, holdings of A-shares and H-shares are not uniquely determined because the shares are ‘redundant’ assets. Yet, the value for consumption, bond positions and price are uniquely determined. In the simulation, we keep agents’ shareholdings of the two assets fixed over time so that they don’t trade stocks with each other.
consume nothing $c^1_M \to 0$. Given $\delta^2$, the smaller $\delta^1$ is, the faster the economy converges to the limit.

9.2.2 Diverse Beliefs

Let us turn to the case where the two types of agents have diverse beliefs on fundamentals. Specifically, type 1 (2) agents’ subjective probability is $\text{prob}^1(D_t)$ ($\text{prob}^2(D_t)$).

Step 1: The first step is to solve the $MRS_{12}$ in the first period $\lambda_0$. It is the same as the step above but the change of measure by multiplying the ratio of the subjective probability to the objective probability should be taken into account.

Step 2: Draw one long series of $M_L$ periods of dividends using the random number generator. Simulate the time-varying $\lambda_t$ following:

$$\lambda_t = \alpha_{t-1}(D_t)\lambda_{t-1}$$

where $\alpha_{t-1}(D_t) = \frac{\text{prob}^2(D_t)}{\text{prob}^1(D_t)}$. Given $\{\lambda_t\}_{t=0}^{M_L}$, find the corresponding consumption $\{C^1_t, C^2_t\}_{t=0}^{T_L}$ by the market-clearing condition. Calculate the present value of primary deficits adjusted by the change of measure for agent 1, $Dd^1_t \equiv \sum_{j=0}^{M_L} \delta^{1+j} \frac{u'(C^1_{t+j})}{u'(C^1_t)} (C^1_{t+j} - D_{t+j}) \frac{\text{prob}(D_t)}{\text{prob}(D_t)}$, which can be solved backward assuming $Dd^1_{M_L} = 0$.

Step 3: Run a regression of $\{Dd^1_t\}_{t=1}^{M_L-O}$ on $\{D_t\}_{t=1}^{M_L-O}$ and $\{\lambda_t\}_{t=1}^{M_L-O}$ for which the last $O$ periods are omitted. The time-varying bond holding positions of type 1 agents are the fitting values of the regression. The bond positions of type 2 agents are the opposite of those of type 1 agents.

We assume $\text{prob}^1(D_t = D^h) = 0.51$, $\text{prob}^1(D_t = D^l) = 0.54$ and $\gamma^1 = \gamma^2 = 1$. Type 1 agents are more accurate in terms of the “distance” of the subjective probability from the true probability. As is seen in Figure ?? for one simulation, they accumulate assets and consume more goods over time while type 2 agents behave in the opposite way. In the limit, type 1 agents will consume the total resources, while type 2 agents get nothing. It is the relative correctness of the perceived

\[\text{In this case, the state-contingent-bond positions inherited in period } t \text{ are not only a function of } D_t \text{ but also a function of } \lambda_{t-1} \text{ i.e. } B_{t-1}^1(D^h) = E(Dd_t^1|D_t = D^h, \lambda_{t-1}) \text{ and } B_{t-1}^1(D^l) = E(Dd_t^1|D_t = D^l, \lambda_{t-1}).\]
beliefs that drives this bond-trading pattern rather than their degree of optimism. While type 2 agents are more optimistic than type 1 agents, it is the type 2 agents who want to purchase more state-contingent bonds. However, although the two agents have heterogeneous perspectives about the economic fundamentals, the prices of the two shares are identical for all states.

9.3 An Assumption for Learning

According to the arguments described by AMN, without strict rational expectations we may obtain \( E^P[C_{t+1}^i] \neq E^P[C_{t+1}] \), even if \( C_{t+1}^i = C_{t+1} \) holds ex-post in the equilibrium. We invoke a similar approximation, as follows:

\[
E^P_t \left( \frac{C_{t+1}^i}{C_t^i} \right)^{-\gamma} (P_{t+1}^j + D_{t+1}) \approx E^P_t \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} (P_{t+1}^j + D_{t+1}), j = A, H
\]  

(18)

The following assumption as in AMN provides sufficient conditions for this to be the case:

**Assumption** We assume that \( Y_t \) is sufficiently large and that \( E^P_t P_{t+1}^j / D_t < M, j = A, H \) for some \( M < \infty \) so that, given finite asset bounds \( S \) and \( \overline{S} \), the approximation (18) holds with sufficient accuracy.

9.4 Differentiable Projection Facility

The function \( \omega \) for the differentiable projection facility is:

\[
\omega(\beta) = \begin{cases} 
\beta & \text{if } x \leq \beta^L \\
\beta^L + \frac{\beta - \beta^L}{\beta^U - \beta^L} (\beta^U - \beta^L) & \text{if } \beta^L < x \leq \beta^U
\end{cases}
\]

In our numerical exercise, we choose \( \beta^U \) such that the implied price-dividend ratio never exceeds \( U^{PD} = 600 \) and set \( \beta^L = \delta^{-1} - 2(\delta^{-1} - \beta^U) \).

9.5 Alternative Initial Beliefs

We can recover the initial beliefs investors hold in November 2014 by substituting the actual A- and H-share prices into the belief updating scheme (15), in the same way as we did in section 5.
The parameter values in (15) are the same as section 4.5. We use the actual stock prices data from April 2014 since this is the time the Chinese government announced the approval of the connect program. Investors are assumed to have RE beliefs at the beginning and then update their beliefs according to (15) after the announcement. The alternative set of beliefs recovered are very close to the ones calibrated in section 4.5. Hence, we have the similar simulated moments reported in Table 11.

<table>
<thead>
<tr>
<th>Targeted Moments</th>
<th>Data</th>
<th>Model (95% interval)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E\left(\frac{P^A}{P^H} * 100\right)$</td>
<td>126.68</td>
<td>[94.84, 130.78]</td>
</tr>
<tr>
<td>$\sigma\left(\frac{P^A}{P^H} * 100\right)$</td>
<td>6.98</td>
<td>[4.41, 26.98]</td>
</tr>
<tr>
<td>$\rho\left(\frac{P^A}{P^H} * 100\right)$</td>
<td>0.88</td>
<td>[0.96, 0.99]</td>
</tr>
<tr>
<td>$corr(P^A_t, P^H_t)$</td>
<td>0.91</td>
<td>[0.57, 0.99]</td>
</tr>
<tr>
<td>$corr(P^A_t - P^H_t, P^H_t)$</td>
<td>0.18</td>
<td>[-0.16, 0.93]</td>
</tr>
<tr>
<td>$\sigma(P^A_t)/\sigma(P^H_t)$</td>
<td>1.20</td>
<td>[1.06, 2.63]</td>
</tr>
</tbody>
</table>

### 9.6 Chinese Stock Market Volatility and Internal Rational Learning

AMN present IR learning asset pricing model can reproduce US stock market volatility. To check whether IR model can explain the Chinese stock market volatility, we estimate the IR learning model with Chinese data in quarterly frequency following AMN. The stock index is Shanghai Composite Index. Table 12 reports estimation outcomes when assuming $\gamma = 5$. The second column in Table 12 reports the asset pricing moments from the data. Columns 3 and 4 report the model moments and t-statistics, respectively. The estimated model quantitatively replicates the mean, standard deviation, and persistence of the PD ratio, as well as the mean and standard deviation of stock return. In addition, Figure ?? shows realizations of the time-series outcomes for the PD ratio generated from simulating the estimated model, for the same number of quarters as the number of observations in our data sample. The simulated time series display price booms and busts.
### Table 12: Estimation Outcomes

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Data</th>
<th>Model</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean PD ratio $E(P/D)$</td>
<td>389.47</td>
<td>229.66</td>
<td>1.66</td>
</tr>
<tr>
<td>Std.dev. $\sigma(P/D)$</td>
<td>268.60</td>
<td>189.27</td>
<td>1.72</td>
</tr>
<tr>
<td>Autocorrel. PD ratio $\rho_{PD,-1}$</td>
<td>0.94</td>
<td>0.90</td>
<td>0.85</td>
</tr>
<tr>
<td>Mean stock return $E(r_s)$</td>
<td>2.60</td>
<td>1.38</td>
<td>1.31</td>
</tr>
<tr>
<td>Std.dev. stock return $\sigma_{r_s}$</td>
<td>16.20</td>
<td>16.85</td>
<td>-0.44</td>
</tr>
<tr>
<td>Mean dividend growth $E(\Delta D/D)$</td>
<td>1.97</td>
<td>1.00</td>
<td>1.26</td>
</tr>
<tr>
<td>Std.dev. dividend growth $\sigma(\Delta D/D)$</td>
<td>6.84</td>
<td>6.84</td>
<td>0</td>
</tr>
<tr>
<td>Discount factor $\hat{\delta}$</td>
<td></td>
<td></td>
<td>0.996</td>
</tr>
<tr>
<td>Gain coefficient $\hat{\alpha}$</td>
<td></td>
<td></td>
<td>0.0032</td>
</tr>
</tbody>
</table>

#### 9.7 Data Sources

We have used financial and macroeconomic data. The financial data including Hang Seng China AH Premium Index, Hang Seng China A(H) index, Shanghai Composite Index, dividend yield, market capitalization, and Northbound (Southbound) trading quota balance are downloaded from the Wind Financial Database (http://www.wind.com.cn). The sample period of Hang Seng China AH Premium, A and H Index is from Jan 2006 to June 2019. The daily (monthly) price series has been transformed into a weekly series by extracting the value of the last trading day of the corresponding week (month). The dividend yield is a moving average of the dividends in the previous 11 months and the current month divided by the Shanghai Composite Index. The monthly dividends on the index portfolio are obtained from the monthly dividend yield under the assumption that dividends have been approximately constant during the last 12 months. We have quarterly dividends by adding up the corresponding monthly series. Following Campbell (2003), we deseasonalize dividends by taking averages of the actual dividend payments over the current and preceding three quarters. The sample period of dividend yield is from 1995 Q1 to 2018 Q4. Stock market turnover data are from CEIC (www.ceicdata.com). The U.S. stock market data are from Shiller’s website (http://www.econ.yale.edu/~shiller/) and CRSP. The spot exchange rate and the forward rates are also from Wind.

The transaction costs and the dividend taxes can be found on the official websites of regulatory authorities and stock exchanges. The transaction costs and the dividend taxes in SSE are found on "http://www.sse.com.cn", while the transaction costs and the dividend taxes in HKSE are in
The macroeconomic data including consumption, 1-year deposit interest rate and CPI in quarterly frequency are downloaded from CQER Fed Atlanta, which are used in Chang et al. (2016). The sample period is from 1995 Q1 to 2018 Q4. To obtain real values, nominal variables are deflated using China CPI.