AH Premium: A Natural Experiment

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Abstract

A large portion of the Chinese stocks is traded both in the Shanghai (A-share) and Hong-Kong (H-share) markets. The A- and H- shares are different assets since they can not be exchanged one-to-one. A-shares have sold at a premium: the AH premium. This premium is large (20-50%) and persistent, it has been present since the two markets were connected in Nov 2014 until now. Since both shares pay the same dividend and traders can operate on both markets this provides a natural experiment to test asset pricing models. We show that various standard Bayesian/RE asset pricing models cannot explain the AH premium, but a model of internally rational learning where agents learn about stock prices provides a natural explanation. This emphasizes the importance of modeling investors who learn about equity prices. The premium survives the introduction of convergence traders: those who bet on the AH-premium going to zero are highly likely to suffer big losses.

Key Words: AH Premium, Asset Pricing, Learning about Prices

JEL: G12, G15

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1 Introduction

The shares of many important Chinese companies trade in both the Shanghai and the Hong Kong markets. Even though these twin shares pay the same dividends and have the same voting rights, they are distinct shares labelled "A-shares" and "H-shares" respectively. They are different assets, they can not be traded one-to-one anywhere. The two markets were connected in November 2014, since then investors in either market can buy both A- and H- shares for all companies participating in the connecting program, we refer to this connected market as the "AH-market".

Surprisingly, A-shares trade at a substantial premium relative to the H-shares of the same companies in the AH-market. This premium increased greatly precisely after the connection date. It has been fluctuating with a mean of 30 percent and a maximum of 50 percent since the connection.[1]

The aim of this paper is to explain the AH premium. Traditional asset pricing theories are unable to explain such a large premium, including theories based on Bayesian learning about fundamentals. However, we find that models of learning about stock prices provide a very natural explanation.

Understanding the AH premium is important, first, because of the huge size of the AH-market: the market value of AH-share reached 2.7 trillion U.S. dollars in 2015, which was about 70% of the market capitalization of the London stock exchange;[2] second, because it speaks to the consequences of financial liberalization; third, because it matters for investors in this market; and fourth, because it provides a rare opportunity for economists to test asset pricing theories (in terms of stock price expectation formation) with a natural experiment.

We have not found a satisfactory explanation of the AH premium in the literature. Some explanations based on "segmented markets" with agents that are heterogeneous across markets have been provided. But the AH-market is not segmented since November 2014, and the premium became very large precisely after the connection date, therefore standard asset pricing theories imply that market forces should equalize prices even if market participants are heterogeneous.

[1] See Figure 2.

[2] See subsection 2.2 for descriptive details of the AH-market.

Other informal explanations appeal to transaction costs and dividend taxes, but as we show these would explain only a small part of the premium.[3] Ad-hoc explanations based on Chinese traders being less sophisticated (granny trading) or lack of competition in the Chinese stock market are even harder to sustain, only sufficiently wealthy traders can participate in the AH-market,[4] therefore most Chinese grannies have a much harder time trading stocks than American grannies.[5] Only systematic research can discover if US or Chinese stock markets are sufficiently well described by the competitive market assumption, along with an appropriate assumption about how agents form expectations.

We show that learning about stock prices provides a natural explanation. Recent papers based on Internal Rationality (IR) such as Adam and Marcet (2011), Adam, Marcet and Nicolini (2016) and Adam, Marcet and Beutel (2017) show that learning about stock prices is compatible with rational investors. They show that learning about prices explains observed volatility of stock prices. Investors' subjective beliefs about stock prices are given by a model that is a good description of actual prices and this perceived model is hard to reject given actual stock prices. Under this framework agents realize that an A-share is actually a different security from an H-share and that it can be purchased or sold next period at a possibly different price. Even if the A-, H- dividends are (nearly) the same, just because investors expect A- and H- prices to differ these prices will actually differ in equilibrium, even when investors are (internally) rational.

When agents learn about stock prices high expected capital gains generate high prices, this leads to high expected capital gains and so on in a self-referential fashion. Our explanation for Figure 2 would be that some factors increased either the actual price and/or the expected capital gains in A-shares, shortly after November 2014 (perhaps the more liquid market, new market participants, the widely spread optimistic media reports on the A-share market, or perhaps some special events in China around those dates). This increased A-prices, leading to high expected

[3] See subsection 2.3 for a more detailed discussion of the literature and section 3 for a detailed analysis of how these explanations do not work for the AH premium.

[4] We discuss this in subsection 2.2.

[5] Another informal explanation has been lack of competitiveness in the Chinese stock market. But this is difficult to square with the fact that A-shares have a higher price. In any case, lack of competition is always a possible explanation for almost anything. The US stock market is not a textbook example of competitiveness either and yet a large asset pricing literature entertains the assumption of competitive markets in American stocks.

prices and so on. We show formally that learning about prices can generate a large and persistent AH premium and we match the key moments of the observed AH premium data.

This explanation of the AH puzzle implies that agents' expectations could be a key element determining the outcome of financial liberalization. The average and the variation of the AH premium were relatively small a couple of years before November 2014, and the market connection took place in part to promote price convergence. But as seen in Figure 2, the result was the opposite, and the AH premium became much larger after connecting the markets. To the extent that the AH premium is seen as undesirable, as it injects uncertainty in the system and it may promote unproductive speculation, our observation should be relevant for the recent connection of the Shanghai and London stock markets.[6]

After Shanghai and Hong Kong markets connected, the widening AH premium motivated some investors to bet on the convergence of prices of A- and H- shares, short selling A-shares and long buying H-shares. Chinese finance newspapers, however, reported in June 2015 that these investors encountered great losses in the end.[7] We show, by simulation, that AH premium caused by investors learning about prices may not shrink in the short term and convergence traders are likely to incur high losses.

Our paper also contributes to the debate on how to model investors' expectations in stock markets. As is well known, it is very difficult to explain stock price volatility and the behavior of survey expectations under rational expectations (RE). A large literature deviates from RE by assuming agents imperfectly know the distribution of fundamental shocks, but this literature assumes that investors understand that prices equal the present-value of dividends, we dub this literature Bayesian/RE,[8] it includes models of Bayesian learning, "agreeing to disagree", robustness and behavioral economics.[9] Although Bayesian/RE can not explain quantitatively stock price

[6] Shanghai-London Stock Connect is a mechanism that connects the London Stock Exchange and the Shanghai Stock Exchange. Eligible companies listed on the two stock exchanges can issue, list and trade depositary receipts on the counterpart's stock market in accordance with the corresponding laws and regulations. The connection took place on June 17, 2019

[7] See the news in Chinese in the link: http://finance.jrj.com.cn/2015/06/11082819343903.shtml

[8] Adam, Marcet and Beutel (2017) dub this literature Bayesian/RE learning. They point out that this literature makes a very asymmetric assumption about fundamentals and prices: investors are assumed not to understand the behavior of fundamentals (say, dividends) but investors understand perfectly well how current stock prices relate to future fundamentals, so that investors are assumed to have RE about the pricing function.

[9] See subsection 2.3 on related literature for more details.

volatility, many authors claim that it could explain survey behavior, just as learning about prices can.[10]

Some researchers have designed lab experiments to distinguish these two theories, where subjects can trade stocks that pay the same dividend, they have a certain understanding of fundamentals and the experiment is designed to understand if agents will quickly converge to a common knowledge of the pricing function (as it is assumed in the Bayesian/RE literature but not in the learning about prices literature). These papers tend to show that laboratory subjects still differ in their price expectations. We contend that the AH-market tests the same issue: to the extent that traders' stakes are many times higher in the true AH-market than in the lab, we think of this as a natural experiment rejecting Bayesian/RE models in favor of learning about prices.

The paper is structured as follows. Section 2 discusses institutional issues of the Chinese stock market and the literature review. Section 3 argues that the standard asset pricing models where agents know the pricing function have difficulties in generating data-like AH Premium. We show that learning about prices can explain the AH premium naturally in section 4. Section 5 shows that convergence traders are likely to suffer big losses. Section 6 discusses some extended price learning model. Section 7 concludes this paper.

2 Overview of the Chinese Stock Markets and Literature Review

This section describes some key features of twin shares in the Chinese stock market and literature related to our paper. We argue, first, that markets for twin shares (both the AB- and the AH-markets) were segmented before Nov 2014, but the AH market was connected after this date. We also argue that the AH premium before Nov 2014 is easy to understand using segmented market models with heterogeneous agents found in the literature, but the large AH premium after that date is a puzzle, specially because the premium increased so much precisely after the connection.

[10] See Adam, Marcet and Beutel (2017).

2.1 Mainland Chinese Stock Market

The Chinese mainland stock market is relatively young, it opened in 1990 with the establishment of the Shanghai Stock Exchange (SSE) and the Shenzhen Stock Exchange (SZSE). Stocks traded on the two exchanges with RMB (Chinese currency) are called A-shares. From 1990 to 2015, the Chinese economy grew at 10% annually on average and this led to the rapid development of equity markets. The market capitalization of the A-shares reached about $8.4 trillion at the end of 2015 and became the second–largest in the world. The value of A-share trading was $47 trillion in 2015, while trading in the US market was $31 trillion.[11]

The Chinese stock market is even more volatile than the U.S. stock market. Table 1 gives some basic statistics of both stock markets. And Figure 1 plots the Shanghai stock price index (Shanghai Composite Index) from 1995 to 2018 in real terms. There were two boom-bust episodes: the stock price index reached its historical peak in 2007 having risen from a trough in 2005, and then it quickly busted. Stock prices boomed again in the second half of 2014 and almost doubled by mid-2015, but still quickly trended down.

A unique feature in the mainland Chinese stock market is that several dozen companies have issued twin shares listed on the same exchanges. Twin shares dubbed A- and B-shares have existed since 1993, they have identical dividend and voting rights, but they were traded by different investors, so it is not surprising that they had different prices. A-shares traded with RMB used to be restricted to mainland investors before November 2014. In addition many companies issued B-shares that were strictly confined to international investors before February 2001 and traded with U.S. dollars. Domestic investors were allowed to trade B-shares using U.S. dollar since February 2001 but, as discussed in Mei, Scheinkman and Xiong (2009), the difficulties for Chinese citizens to acquire US dollars still serve as severe restrictions for domestic investors to participate in the B-share market. Therefore, A-share market and B-share market were strictly segmented before February 2001 and are still segmented to a large extent for Chinese investors. The literature documented that A-shares traded at higher price than B-shares despite their identical rights, e.g. Fernald and Rogers (2002) and Mei, Scheinkman and Xiong (2009). It is easy to explain this

[11] See Allen et al. (2017).

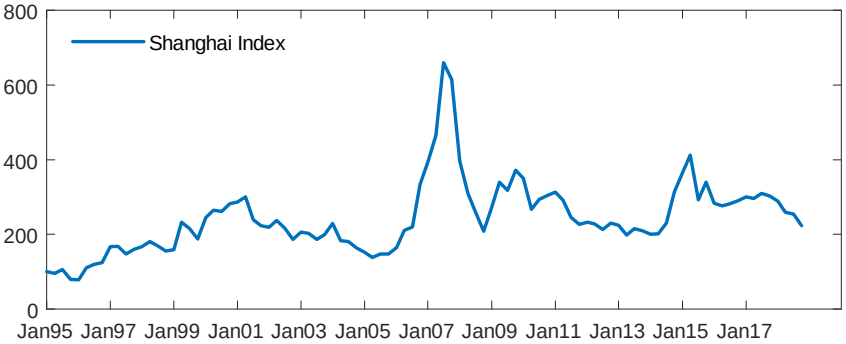


Figure 1: Real Stock Price Index in Shanghai Exchange

Statistics	SSE	SZSE	S&P500	CRSP
Std.dev. stock return σ_{rs}	16.46%	18.73%	8.17%	8.69%
Std.dev. stock return σ_{PD}	227.62	148.45	47.26	54.94
Std.dev. dividend growth $\sigma_{\Delta D/D}$	1.81%	0.82%	0.86%	0.86%

Table 1: China and U.S. Stock Market Volatility in Real Terms, Quarterly Frequency, Sample Period: 2000:1 - 2015:4

premium in the light of models of segmented markets and heterogeneous agents.

2.2 AH-shares and Shanghai-Hong Kong Stock Connect

The shares of companies incorporated in mainland China but traded on the Hong Kong Stock Exchange (HKSE) with Hong Kong dollars are called H-shares. Currently, about one hundred mainland China companies dual-list in both A-shares (most on the Shanghai stock exchange) and H-share markets. Twin stocks of the same companies have the same fundamentals since they are identical with respect to shareholder rights, such as voting and profit-sharing, but it is impossible to buy one class of share and exchange it for another class of share at one-to-one price, as they are nominally different stocks exchanged in different markets.

Most of the large companies are included in the AH market: the market value of A-shares of these dual-listed companies accounts for 20% of the total A-share market capitalization, while the market value of H-shares of these companies accounts for 35% of the market capitalization of HKSE. These shares are very liquid.

Before November 17, 2014, the Shanghai and Hong Kong markets were segmented: mainland investors were not allowed to trade in the H-share market and foreign investors were barred from

investing in A-share market. Therefore an AH premium before November 2014 can be explained by appealing to segmented markets with heterogeneous agents. But the Shanghai and Hong Kong markets became connected after the Shanghai-Hong Kong Stock Connect program was established on November 17, 2014. Through the connect program, mainland investors can trade more than 300 stocks on the HKSE and foreign investors can trade more than 500 stocks on the SSE.[12] All of the Shanghai and Hong Kong dual-listed companies are included in the connect program. Mainland investors can participate in the H-share market through the trading and clearing facilities of their home exchange, as can Hong Kong and international investors in the Shanghai market.[13] Therefore the A- and H- markets are not segmented since then. Transaction costs in each market are low and the same across the investors as we quantify in section 4. Trading and settlement currency through Shanghai-Hong Kong stock connect is RMB. Mainland investors do not need Hong Kong dollars to buy H-shares.[14] Hong Kong and international investors can acquire RMB easily in the offshore RMB market to buy A-shares.

Surprisingly, the AH premium increased markedly after the connection date. The Hang Seng China AH Premium Index plotted in Figure 2 measures the weighted average price ratio (price of A-shares over price of H-shares of same companies) in percentage for these AH-shares.[15] The index of 100 means that A-shares are trading at par, an index larger than 100 indicates A-shares trade at a premium. After the market connection, according to standard asset pricing theories, the AH premium index should have converged to 100 (or to somewhere close to 100 because of the different dividend taxes and the transaction costs) even under heterogeneous agents. However, it diverged dramatically to almost 150 at peak in June 2015 and it fluctuated between 120 and 150 since

[12] See Ma, Rogers and Zhou (2019).

[13] Precisely, all Hong Kong and international investors are allowed to trade A-shares listed in Shanghai. All mainland institutional investors and individual investors who have at least RMB500,000 in their investment and cash accounts are eligible to trade H-shares. According to the Financial Times report in July 2015, small retail investors hold less than 5% of the overall market value of A-shares (https://www.ft.com/content/f3d94f92-2715-11e5-9c4e-a775d2b173ca).

[14] China Securities Depository and Clearing Company provides unlimited currency conversion service automatically in the background for the mainland investors to trade in the H-share market. After mainland investors sell H-share stocks, they get RMB rather than Hong Kong dollars.

[15] Let P^A be the weighted average price of A-shares and P^H the weighted average price of H-shares, the index is $P^A/P^H * 100$. The exchange rate is taken into account when constructing this index. The index methodology refers to https://www.hsi.com.hk/static/uploads/contents/en/dl_centre/methodologies/IM_chinaahe.pdf

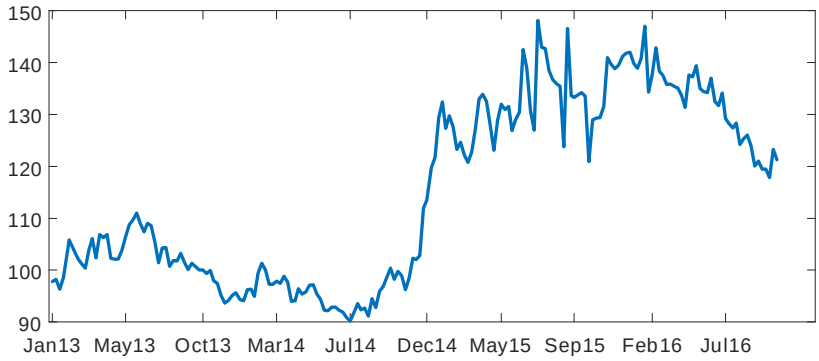


Figure 2: Hang Seng China AH Premium Index

then.[16] For the cross-sectional individual stocks, 97 out of 101 individual stocks have premium for the A-share over the H-share after the connection. It is not just one or two big companies with large weights that account for all of the AH Premium.[17] Note, however, that A-share and H-share prices are highly positively correlated, but not perfectly correlated, as shown in Figure 3: from November 2014 to June 2015, both A-share and H-share prices rose but A-share prices rose faster than H-share contributing to the divergence in prices. A similar pattern of different speeds of adjustment is observed when the price indexes fell after July 2015.

Some economists and market participants have expressed the view that the Chinese central government directly and frequently intervenes in the mainland stock market. However, it is not clear that it does more so than the US government. Since 2005, it has only intervened once when the A-share stock market bubble burst at the end of June 2015. The Chinese government, out of concerns about the high leverage taken by many Chinese investors, required a national team of state-owned security companies to support stock prices by adopting long positions to avoid a severe financial crisis.[18] It is common worldwide that governments directly intervene to stabilize the financial market turmoils, for example, the US government intervention (Paulson's plan) in 2008 (Veronesi and Zingales, 2010). The intervention of Hong Kong government in the stock market during the 1998 Asian financial crisis is also well-documented.[19]

[16] The AH premium index is still high at 130 on August 2019.

[17] The weight of each firm in the AH Premium Index can be found in https://www.hsi.com.hk/static/uploads/contents/en/dl_centre/factsheets/chinaahe.pdf.

[18] See Huang, Miao and Wang (2019).

[19] See Goodhart, Lu and Goodhart (2003).

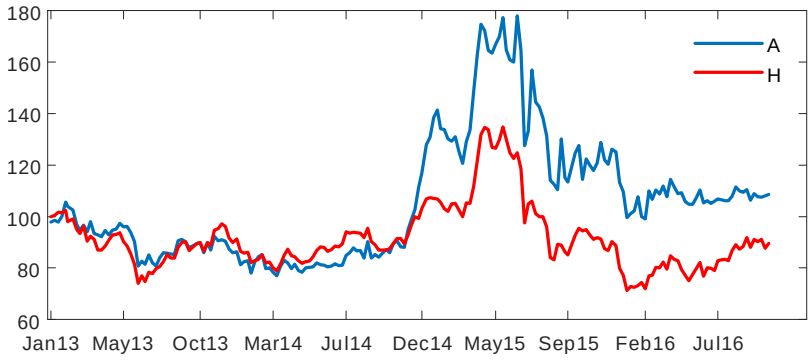


Figure 3: Hang Seng China AH-A and AH-H Price Index

2.3 Related Literature

This paper is related to the literature on the price differences of twin stocks. First, Froot and Dabora (1999), Fernald and Rogers (2002) and Mei, Scheinkman and Xiong (2009) show that transaction costs are not the main reason for price differences. Second, many papers focus on explaining the price differences in the segmented market environment, where AB premium is a prominent example. Heterogeneities among different participants across segmented markets such as agents' different stochastic discount factors (Fernald and Rogers 2002), and diverse beliefs about fundamentals (Chan, Menkveld and Yang 2008; Mei, Scheinkman and Xiong 2009; Jia, Wang and Xiong 2017) are argued to explain the price differences. However, heterogeneities fail in explaining the price differences after markets connection in Nov 2014. The difficulties of explaining price differences in connected markets are also widely documented in the literature. For instance, Lamont and Thaler (2003) mention the price differences of twin stocks for three different firms in the connected US and European markets,[20] Froot and Dabora (1999) mention that this is an anomaly difficult to be rationalized. The AH premium is arguably more puzzling because of the much larger size of the market compared with those considered in the papers just mentioned. To our knowledge, our paper is the first one to propose a micro-founded asset pricing model that can rationalize and quantitatively explain these premia in twin stocks and, in particular, the AH premium.

As is well known, external habit (Campbell and Cochrane 1999) and long run risk (Bansal

[20] They are Royal Dutch Petroleum and Shell Transport and Trading, PLC; Unilever N.V. and Unilever PLC; and SmithKline Beecham.

and Yaron 2004) can explain the stock market regularities under RE but some difficulties of these theories have been pointed out and these papers in any case can not explain survey behavior. A large literature deviates from RE by assuming agents imperfectly know the fundamentals, but this Bayesian/RE literature assumes that investors understand that prices equal the present-value of dividends. It includes models of Bayesian learning (e.g. Timmermann (1996), Collin-Dufresne, Johannes and Lochstoer (2016)), agreeing to disagree (e.g. Scheinkman and Xiong (2003), Ehling, Graniero and Heyerdahl-Larsen (2018)), robustness (e.g. Hansen, Sargent, and Tallarini (1999), Anderson, Sargent and Hansen (2003)), behavioral economics (e.g. Barberis, Shleifer and Vishny (1998) and Barberis, Greenwood, Jin and Shleifer (2015)). However, these can not provide an explanation for the AH-premium, since equilibrium prices for stocks that pay the same dividend are equalized in equilibrium when markets are connected, even in the presence of heterogeneous agents.

To rationalize the AH premium in connected markets (also the price differences of twin stocks in general), it may be necessary to relax agents' knowledge of the equilibrium pricing function, as is done in adaptive learning models. Examples of this type of models include Lansing (2010), Branch and Evans (2011), Boswijk, Hommes and Manza (2007), Carceles-Poveda and Giannitsarou (2008), all of which build asset-pricing models where agents learn adaptively and don't have the perfect knowledge about the true stochastic process for payoff relevant variables beyond their control. Adam and Marcet (2011) develop 'internal rationality' to provide microfoundations for models of adaptive learning. Adam, Kuang and Marcet (2012) show an application to housing prices, Adam, Marcet and Nicolini (2016) explain stock price volatility, Winkler (2019) matches both asset price and business cycle moments.

The experimental literature also provides evidence supporting the presence of subjective price beliefs. Hirota and Sunder (2007) and Asparouhova et al. (2016) design a Lucas asset pricing laboratory experiment, they find that stock prices display excess volatility unaccounted for by fundamentals and it is most likely attributed to participants' subjective price expectations. Crockett et al. (2018) suggest that speculative behavior in the lab causes the emergence of price bubbles.

Recent papers propose various strategies to test expectations-based asset pricing theories. Greenwood and Shleifer (2014), Adam, Marcet and Beutel (2017) and Kuang, Zhang and Zhang (2019) use survey expectations data to test the expectation formations in many asset pricing models. Bansal, Gallant, and Tauchen (2007), Beeler and Campbell (2012), Bansal, Kiku, and Yaron (2012), and Barro and Jin (2016) apply moment matching methods to comparing the empirical performance of habit, long run risks, and rare disasters models. Aldrich and Gallant (2011) utilize a Bayesian framework to compare habit, long run risks, and prospect theory. Our paper instead uses the AH premium in connected markets as a natural experiment to directly test various asset pricing models.

3 AH-Premium with Price Knowledge

There has not been a theoretical study of the features that could generate a premium in twin stocks. In this section we discuss various models to elicit whether available theories where agents know the pricing function (that is, RE or Bayesian/RE) are able to explain the observations. From now on we take it as given in the paper that post-Nov 2014 the AH-market is not segmented, traders in both markets can purchase stocks in the other market.

Some authors have argued that heterogeneity can be important in pricing equity and explaining price differences in twin stocks. Heterogeneity can be due to features such as different stochastic discount factors (Fernald and Rogers 2002), and diverse beliefs about fundamentals (Chan, Menkveld and Yang 2008; Mei, Scheinkman and Xiong 2009; Jia, Wang and Xiong 2017). These issues may indeed be present in the Chinese market. Song, Storesletten and Zilibotti (2011) show that the yearly discount factor implied by the Chinese high saving rates is 0.997, larger than the yearly U.S. discount factor of 0.96. The same is true for heterogeneous expectations. Malmendier and Nagel (2011) argue that agents' expectations are highly determined by the economic conditions they have experienced during their lifetime. Applying this idea to the AH premium we see that during the sample period investors from U.S. and Europe had just experienced the Great Recession while Chinese economy grew substantially, hence one could think that international investors are likely to be more pessimistic than Chinese investors.

Heterogeneity and Bayesian/RE can explain different stock prices in segmented markets. But in connected markets it can only explain different consumption allocations and bond positions, it cannot explain price differences in the two connected markets. To demonstrate this we consider two types of infinitely-lived agents, with type 1 agents standing for mainland investors and type 2 agents for Hong Kong and international investors. Within each group, the agents are homogeneous with preference satisfying Inada conditions. As is common in the Bayesian/RE literature agents understand the pricing function mapping fundamentals into prices, but agents may have expectations about fundamentals that differ from the objective expectations. It is common knowledge that one unit of A-share and H-share pay the same dividends. All agents can participate in the two connected markets but are confronted with a short-selling constraint for stocks. We consider dividends taxes and other transaction costs in the subsection 3.2. Ponzi schemes and rational bubbles are ruled out.

We first show analytically how Bayesian/RE fails to produce any price differences in a complete market model and then extend it to incomplete markets.

3.1 Models with Price Knowledge

This section presents results that follow directly from the literature on heterogeneous agents and "agreeing-to-disagree". Informal explanations of the AH premium are often based only on heterogeneity, we find it useful to explain that heterogeneity alone does not generate a premium.[21] We first present complete market Lucas tree models with two types of agents. Agents have rational expectations in the first case but have heterogeneous expectations about fundamentals in the second case.

In the first case, type i agents of μ^i fraction have utility u^i and subjective discount factor δ^i, for $i \in \{1, 2\}$, represent a fraction $\mu^i > 0$ of the population. Investors' portfolios include A-shares, H-shares, and Arrow securities contingent on dividend realizations. The two types of equities share the same dividend payments *i.e.* $D_t^A = D_t^H = D_t$. The exogenous process of dividend D_t is as simple

[21] For example, see articles in several financial newspapers (in Chinese) https://finance.qq.com/a/20120109/001083.htm. ,https://finance.sina.cn/stock/relnews/hk/2019-10-17/detail-iicezuev2751039.d.html?from=wap,
http://www.xinhuanet.com/finance/2017-02/22/c_129489122.htm.

as possible to make the discussion as transparent as possible. The exogenous dividend process D_t is assumed to be $i.i.d$ taking two values $D^h > D^l$, with $Prob(D^l) = \pi, Prob(D^h) = 1 - \pi$. Arrow security $B_t(D^h)$ or $B_t(D^l)$ are purchased each period, they deliver 1 unit of consumption if the dividend is high or low in period $t+1$. In addition to the dividends, agents also receive an endowment Y_t.[22] Rational agents choose their consumption and asset portfolio optimally to maximize their expected discounted utility subject to budget constraints.

$$\max \sum_{t=0}^{\infty} E_0 \delta^i u^i(C_t^i)$$

$$\begin{aligned} s.t. \qquad & S_t^{i,A} P_t^A + S_t^{i,H} P_t^H + C_t^i + B_t^i(D_h) Q_t(D_h) + B_t^i(D_l) Q_t(D_l) \\ = \quad & S_{t-1}^{i,A}(P_t^A + D_t^A) + S_{t-1}^{i,H}(P_t^H + D_t^H) + B_{t-1}^i(D_t) + Y_t, \ \forall t \end{aligned}$$

and bounds on shareholding

$$\begin{aligned} \underline{S} \ &\leq \ S_t^{A,i} \leq \overline{S} \\ \underline{S} \ &\leq \ S_t^{H,i} \leq \overline{S}. \end{aligned}$$

The upper and lower bounds will be common to the remainder of the paper.[23] Throughout the paper, the aggregate supply of both shares are 1 such that market clearing conditions of shares read as

$$\mu^1 S_t^{1,Z} + \mu^2 S_t^{2,Z} = 1 \ \ \forall Z = A, H.$$

The budget constraint and stocks market clearing condition also hint that the two types of shares are actually distinct. It is forbidden to sell A-share in the market of H-share and vice versa. For more model details refer to Appendix A.1.

[22] Y_t is assuemd to be constant (or deterministic) without loss of generality.

[23] As is usually assumed in the literature, the lower bound is more stringent than the upper bound.

The standard full-insurance condition holds in equilibrium

$$\delta^1 \frac{u_c^1(C_{t+1}^1)}{u_c^1(C_t^1)} = \delta^2 \frac{u_c^2(C_{t+1}^2)}{u_c^2(C_t^2)}, \tag{1}$$

where u_c^i is the marginal utility for type i agents.

This condition implies that there exists a unique market stochastic discount factor (SDF), therefore using either agent's SDF to price any asset leads to the same answer. Since agents are assumed to know the pricing mapping from the fundamentals to prices — the present-value form of equity prices, using type 1 agent's SDF, we have

$$\begin{aligned} P_t^A &= E_t[\sum_{j=1}^{\infty} (\delta^1)^j \frac{u_c^1(C_{t+j}^1)}{u_c^1(C_t^1)} D_{t+j}^A] \\ &= E_t[\sum_{j=1}^{\infty} (\delta^1)^j \frac{u_c^1(C_{t+j}^1)}{u_c^1(C_t^1)} D_{t+j}^H] \\ &= P_t^H. \end{aligned} \tag{2}$$

Therefore we have zero AH premium even with heterogeneous agents.

Some authors have documented that foreign investors tend to be pessimistic about the Chinese economy, while mainland Chinese citizens have more optimistic views (e.g. Jia, Wang and Xiong 2017), or that mainland Chinese citizens have an informational advantage about fundamentals than foreigners (e.g. Chan, Menkveld and Yang 2008). Stockbrokers and market analysts tend to propagate this kind of story as a way to rationalize the AH premium.

To analyze the potential for this explanation under a Bayesian/RE model, we consider our second case where two types of agents do not know the objective probability π. Specifically, we assume type 1 agents perceive $Prob^1(D^h) = w$, while for type 2 agents $Prob^2(D^h) = v$. Heterogeneous beliefs are common knowledge for all agents but they agree to disagree. To isolate the effects of diverse beliefs, heterogeneities in discount factors and utility function are abstracted from.

In this case, the first-order conditions with respect to the Arrow securities lead to

$$\begin{aligned}
\delta \frac{u_c(C^1_{t+1}(D^h))}{u_c(C^1_t)} w &= \delta \frac{u_c(C^2_{t+1}(D^h))}{u_c(C^2_t)} v \\
\delta \frac{u_c(C^1_{t+1}(D^l))}{u_c(C^1_t)} (1-w) &= \delta \frac{u_c(C^2_{t+1}(D^l))}{u_c(C^2_t)} (1-v)
\end{aligned}$$

which shows that the SDF multiplied by the subjective probability across the two agents are identical in each state. Therefore the price evaluated with agent 1's SDF and subjective probability equal to that evaluated with agent 2's SDF and subjective probability. Any agent's SDF can be used to price equity. Therefore, an equation similar to (2) but with subjective expectations still applies, and the conclusion of no premium applies also with diverse beliefs.

In fact, heterogeneities affect the allocation of consumption and bonds positions. For instance, higher risk aversion of agents induces smoother consumption, a larger subjective discount factor and more precise subjective belief about fundamentals let the agents accumulate assets and achieve dominance in the economy. This is demonstrated in Appendix A.2 by simulation. However, we still have a zero AH premium.

The model with Arrow securities served to provide the basic logic with the aid of analytical expression on why the heterogeneities across investors are not relevant for AH-premium after connection. But the existence of the Arrow securities seems to be an extreme assumption. We provide the following argument for a lack of price differences in the incomplete market models when agents have the price knowledge. In those models, even though the marginal persons could switch across the two types of agents from time to time in the absence of the Arrow securities, the present value formula of prices always hold.[24] Therefore there exists one equilibrium where both prices of A-share and of H-share are equal to the present value of dividends and prices across the two types of shares are equalized.[25] The intuition is that the two types of shares are perfect substitutes for any agent with the price knowledge (they could learn about dividends due to the

[24] With the common knowledge about each agent's preference and belief, agents know who is the marginal person in the current period and have the right expectation about who is going to be the marginal person in the future.

[25] This is irrespective of any types of SDF^i such as the well-known habit utility described by Campbell and Cochrane (1999) and the Epstein-Zin preference used in Bansal and Yaron (2004) and any types of dividend expectations such as Scheinkman and Xiong (2003) and Barberis et al. (2015).

imperfect knowledge about dividends) despite of the fact that investors could be heterogeneous in many aspects. In addition, for each agent, his expected capital gains across the two types of shares are also identical because they are governed by the same fundamental as a result of the knowledge of price mapping. In these models with price knowledge, there is no room for investors to have different expected capital gains to generate price differences in the equilibrium.

3.2 Adding Dividend Taxes

The real word is not free of trading frictions such as dividend taxes and transaction costs. The transaction costs levied on trading value consisting of stamping duty, security management fee, transfer fee are very tiny and similar across the two markets.[26] However, the dividend taxes are very prominent. According to financial regulations, mainland investors pay a constant 20% dividend taxes for H-shares while Hong Kong and other international investors face a 10% dividend tax for both shares. Dividend taxes for mainland Chinese investors to trade A-shares depend on the holding period. The dividend tax is 5%, 10%, and 20% respectively for investors holding the shares for less than 1 month, between 1 month and 1 year, and more than 1 year respectively.[27] We ignore the transaction costs because of their tiny values but want to explore the effects of dividend taxes on the price premium by adding dividend taxes into the model discussed in section 3.1. Although different dividend taxes across the two markets can bring about some price differences, we will argue that they can not explain some basic features of the observed AH-premium.

Let $\tau^{i,A}$ $\tau^{i,H}$ represent the dividend taxes confronted by type i agents in A-share market and H-share market respectively. Given the above numbers, the after-tax dividend payments per share is $(1-\tau^{1,A})D_t$ and $(1-\tau^{1,H})D_t$ for type 1 agents in each market while it is $(1-\tau^{2,A})D_t$ and $(1-\tau^{2,H})D_t$ for type 2 agents where $\tau^{1,H}=20\%$, $\tau^{2,A}=\tau^{2,H}=10\%$. We choose a 5% dividend tax for type 1 agents to trade A-shares *i.e.* $\tau^{1,A}=5\%$ to give the model the best chance to explain AH premium.

Given the values for dividend taxes and transaction costs mentioned above we have $(1-$

[26] The total transaction cost in the Hong Kong exchange is about 0.118% according to regulations, while it is about 0.169% in Shanghai exchange.

[27] Refer to Appendix A.6 for the data sources of transaction costs and dividend taxes.

$\tau^{1,A})D_t > (1-\tau^{2,A})D_t$ and $(1-\tau^{2,H})D_t > (1-\tau^{1,H})D_t$ in each period. It is easy to prove that, with contingent bonds, an equilibrium is as follows: agent 1(2) buys all the stock in the A(H)-share in the first period and he is against the short-selling constraint on the other share, each agent keeps forever this stock position.[28] This is the equilibrium because of the short-selling constraint and the same SDF across the two agents. The Euler equations for A-shares (H-shares) of the two types of agents can't hold at the same time. It must be that the Euler equation of one agent holds with the other agent against his borrowing constraint. Due to the tax advantage of agent 1(2) on A(H) shares, it is the agent 1(2) that is marginal for A-shares (H-shares).

$$P_t^A = E_t \sum_{j=1}^{\infty} \delta^j \frac{u_c(C_{t+j}^1)}{u_c(C_t^1)}(1-\tau^{1,A})D_{t+j} \tag{3}$$

$$P_t^H = E_t \sum_{j=1}^{\infty} \delta^j \frac{u_c(C_{t+j}^2)}{u_c(C_t^2)}(1-\tau^{2,H})D_{t+j}. \tag{4}$$

Given that (due to complete markets) the SDF of the two types of agents are identical in the economy with state-contingent bonds we have that

$$\frac{P_t^A}{P_t^H} = \frac{1-\tau^{1,A}}{1-\tau^{2,H}}. \tag{5}$$

Therefore differences in price are driven by the gap in dividend taxes confronting the agents agent across the two markets. The gap is constant over time at the value of 105.6%. This is inconsistent with the size of the observed AH premium (recall, between 120% and 150%) and the fact that it fluctuates strongly over time.

The argument so far in this subsection relied on the existence of contingent bonds, consider now the case of the incomplete markets. In this case type 1(2) will hold H(A) shares with positive probability, in other words the agents may invest in both markets in order to accumulate precautionary savings. The exact dynamics of this incomplete market model will depend on exactly the process for income, the type of heterogeneity, the assets available etc, and the result can only found by simulation. However, in many setups, due to the dividend tax advantage, it will still be

[28] An online appendix for the proof will be updated later.

the case that with probability one agent 1(2) holds A(H) shares in all periods in the stationary equilibrium, that is, each agent is marginal in "his" market with probability one, although he may also be marginal in the "other" market in some periods. Under these circumstances equations (3) or (4) still hold, since the Euler equation for the "own" market always holds.

Now, as is well known the SDF are *not* equalized across agents under incomplete markets in all periods, so we can not derive (5) exactly as we did before. However, many papers in the incomplete markets literature report that equilibrium prices are set so as to make SDF across agents *almost* equal in incomplete market equilibria, so that we have $\frac{u_c(C^1_{t+1})}{u_c(C^1_t)} \approx \frac{u_c(C^2_{t+1})}{u_c(C^2_t)}$.[29] (new footnote) Combining these two observations we have that (5) holds approximately. Since, as we argued above (5) is so far from explaining the AH premium, we conclude that the observed premium is incompatible with incomplete markets and RE.

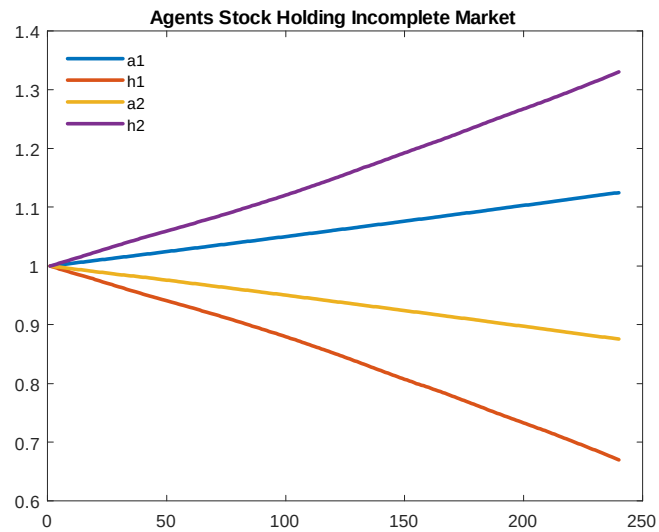
We also simulated a calibrated incomplete market model with A-share and H-share but without the Arrow securities. There are two main differences from the model of the complete market we have discussed. First, the simple discrete dividend process is no longer appropriate for the incomplete market. Following Campbell and Cochrane (1999), dividend growth rates follow $i.i.d.$ lognormal processes

$$\frac{D_t}{D_{t-1}} = a\epsilon^d_t, \log \epsilon^d_t \sim iiN(-\frac{s^2_d}{2}, s^2_d)$$

The endowment growth rate has a mean of a, and the standard deviations of s_d.[30] This dividend endowment setup will be common to the remainder of the paper. Second, having shown that the different discount factors and risk aversions across the two types of agents are not relevant for the premium, we abstract from those heterogeneities to isolate the effects of dividend taxes. Note that the dividend taxes here also cause heterogeneity across the agents.

[29] An early paper reporting a high correlation of SDF's in a two agent model with two assets and heterogeneous incomes is Marcet and Singleton (1999) (see their Table 2 reporting a high correlation of individual consumption and aggregate income for calibrations of Models 1-3 with high serial correlation of income). Levine and Zame (2001) argue that for high discount factors incomplete market equilibria nearly complete the markets. In large OLG models young agents are against a borrowing constraint for a long time so in that model their sdf is not highly correlated with that of old agents, but large differences in age do not seem to be an issue in the AH-market. Some recent models argue that different liquidity can also drive a wedge between agents' SDF's.We do not address issues of liquidity in this paper except to point out that liquidity is very high both in the A- and H- markets, see our comment in subsection 3.3.

[30] The calibration is the same as we do in section 4. Refer to section 4 for the calibration.



The assets are split equally among the two agents initially. The budget constraint with the dividend tax facing the two types of agents are

$$C_t^i + P_t^A S_t^{A,i} + P_t^H S_t^{H,i} = (P_t^A + (1-\tau^{A,i})D_t)S_{t-1}^{A,i} + (P_t^H + (1-\tau^{H,i})D_t)S_{t-1}^{H,i} + Y_t$$

for $i \in \{1, 2\}$.

We implement a short-run simulation and find that over the transition the two types of shares have the same price and both types of agents are marginal for the two types of shares.[31] The dynamics of share-holding are in Figure 3.2 which shows that over the transition type 1 agents accumulate A-shares while type 2 agents accumulate H-shares. This is because agent 1 has a tax advantage over A-shares while agent 2 has a tax advantage over H-shares.

3.3 A Discussion

In sections 3.1 and 3.2 we considered models for which there is a well developed theory and where we could use well known results. Obviously the above analysis does not consider all possible theories where knowledge of pricing function is preserved. We now discuss informally other possible explanations for the AH premium in this subsection. While more research may show that these

[31] Without loss of generality, Y_t is assumed to be constant in the simulation. The simulation method similar to the one shown in Marcet and Singleton (1999) is used. An online appendix for the method will be updated later.

can explain some of the differences in stock prices we will argue that it is unlikely that they can make up for the large and persistent premium that is observed.

Exchange rate risk may play a role in generating AH premium because A-shares are traded in RMB, and H-shares are traded in Hong Kong dollar. We conjecture that a no-arbitrage condition in the stock market similar to the interest rate parity would hold, that is, the expected return differential between A-shares in RMB and H-share in Hong Kong dollar should equal the expected change of the exchange rate. The Hong Kong Dollar was expected to appreciate against the RMB by an average of 0.03%, as measured by the 1-week exchange rate forward from November 2014 to October 2016.[32] According to this no-arbitrage condition, the expected appreciation of the Hong Kong Dollar should have implied an averaged higher H-share price than A-share.

We have not formally explored models of different liquidity. One popular measure of liquidity is the proportion of price-change days for a stock over a certain period (Mei, Scheinkman and Xiong 2009). Based on daily data for the period 2006-2016, the proportion of trading days with price changes for A-shares is 99.35%, while the corresponding proportion for H-shares is 98.95%. And the proportion of trading days with price changes for the U.S. S&P 500 is 96.85% over the same sample periods. This suggests that both A- and H-shares are quite liquid, and A-shares are just marginally more liquid than H-shares. In addition, there is no well accepted model of liquidity in asset markets, therefore exploring this issue formally would be a large undertaking that goes beyond the scope of this paper. We leave a formal study for future research.

There has been a recent interest in rational bubbles in stock markets. For example, Martin and Ventura (2012) in two-period OLG models and Miao and Wang (2018) in infinitely lived agents with transaction costs. Rational bubbles in this literature facilitate liquidity and relax borrowing constraints. The work of Miao and Wang is for infinitely lived agent models and it has the potential of generating bubbles that are quantitatively relevant. Indeed, there could be equilibria in our model where there is a rational bubble in one stock and not on the other thus justifying an AH-premium. However, we know of no papers studying rational bubbles in two

[32] The forward exchange rate can be regarded as investors' expectations about future exchange rate. The expected change is calculated by dividing the forward exchange rate by the spot exchange rate and taking the average over the sample period. Hong Kong dollar was expected to appreciate on average by 0.2%, 0.6% and 1.8% measured by 1-month, 3-month, and 1-year forward exchange rate.

stocks sharing the same fundamentals in infinitely-lived agent models. We conjecture that it is likely that if such an equilibrium was found it would display an explosive AH-premium, unlike the large and stable premium found in the data, but a more formal study of the issue would be of interest.

4 An Internal Rationality Learning Model

Our conclusion from section 3 is that staying within the standard paradigm of agents that know the pricing function has difficulty explaining quantitatively the observed behavior of the AH premium.

This section shows that a parsimonious "internal rationality" learning model, in which agents do not know the pricing function, can generate the AH premium. Agents learn about the underlying rate of stock price growth and, in that way, a difference in prices feeds on itself to generate an AH-premium. This behavior is compatible with rational investors.

As in Adam and Marcet (2011), when the preferences and beliefs of agents are not common knowledge, agents cannot deduce the equilibrium mapping from their own optimization conditions;[33] agents' subjective expectations about future capital gains will not be anchored by their expected future dividends and will be crucial in pricing an equity. We first present the model environment and show that internally rational learning has the potential to generate price differences for dual-listed shares. We then estimate the model parameters to match moments of the AH premium.

4.1 Model Environment

The environment is the same as the incomplete market model of subsection 3.2 except that we assume away dividend taxes to isolate the role of learning and consider a richer process for the endowment Y_t for quantitative purpose.[34] Denote the aggregate consumption supply by C_t.

[33] As is shown in Adam, Marcet and Niconoli (2016) and Adam, Marcet and Beutel (2017), even though agents are homogeneous, this is not the common knowledge and one agent could believe that others would have very different beliefs from them. We abstract from investors heterogeneities. Heterogeneity may be an interesting way to complement the main story of this section but the paper argues learning about prices is the main issue.

[34] The standard process of Y_t is a good description of actual output and allow the consumption supply and dividend have a low correlation. The specific form of Y_t is not crucial for the AH premium.

Total supply of consumption goods in the economy is then given by the feasibility constraint $C_t = Y_t + D_t$. Following Campbell and Cochrane (1999), we impose assumptions directly on aggregate consumption supply

$$\frac{C_t}{C_{t-1}} = a\epsilon_t^c, \log \epsilon_t^c \sim iiN(-\frac{s_c^2}{2}, s_c^2).$$

Aggregate consumption growth follow $i.i.d.$ log-normal processes. The endowment and consumption growth rates share the same mean a, and $(\log \epsilon_t^d, \log \epsilon_t^c)$ are jointly normally distributed with a correlation of $\rho_{c,d}$, and the standard deviations of s_d and s_c.

The economy is populated by a unit mass of infinite-horizon agents. We model each agent $i \in [0, 1]$ to have the same standard time-separable CRRA utility function and the same subjective beliefs. This, however, is not common knowledge among agents, which differs from the previous models. The specification of agent i's expected lifetime utility function is

$$E_0^{\mathcal{P}} \sum_{t=0}^{\infty} \delta^t \frac{(C_t^i)^{1-\gamma}}{1-\gamma}$$

where C_t^i is the consumption profile of agent i, δ denotes the discount factor of time, and γ is the risk-aversion parameter. Note that C_t is aggregate consumption and therefore exogenous in this endowment economy, while C_t^i is endogenous to the model. Instead of the objective probability measure expectations are formed using the subjective probability measure $\mathcal{P}$, which describes the probability distributions for all "external" variables. Section 4.2 contains more details about the probability space.

4.2 Probability Space

This section explicitly describes the general joint-probability space of the variables that investors take as given ("external" variables). Agents are assumed to know the true processes of aggregate consumption supply and dividend growth. In the models discussed in section 3 agents know the pricing function (both under RE and Bayesian RE setups), therefore stock prices P_t^A and P_t^H are not part of the agents' state space, stock prices in that case only carry redundant informa-

tion. However, if agents do not know the mapping because of no common knowledge on agents' preferences and beliefs (see Adam and Marcet 2011) then each competitive agent considers the joint processes of endowment, dividends, and stock prices $\{Y_t, D_t, P_t^A, P_t^H\}$ as external to their decision-making process. Since prices impact the budget constraint of agents, optimal behavior means that equilibrium stock prices of P_t^A and P_t^H should be included in the underlying state space of the investors' decision problem. Formally, we define the probability space as $(\mathcal{P}, \mathcal{B}, \Omega)$ with $\mathcal{B}$ denoting the corresponding σ-Algebra of the Borel subsets of Ω and $\mathcal{P}$ denoting the agent's subjective probability measure over $(\mathcal{B}, \Omega)$. The state space Ω for the realized exogenous variables is

$$\Omega = \Omega_Y \times \Omega_D \times \Omega_{P^A} \times \Omega_{P^H}$$

where Ω_X represents the state space for all possible infinite sequences of the variable $X \in \{Y, D, P^A, P^H\}$. Thereby, a specific element in the set Ω represents an infinite sequence $\omega = \{Y_t, D_t, P_t^A, P_t^H\}_{t=0}^{\infty}$. Agent i chooses plans for the endogenous variables C_t^i, $S_t^{A,i}$, $S_t^{H,i}$ contingent on future realizations of Ω^t, where Ω^t represents the set of histories from period zero up to period t, that is

$$(C_t^i, S_t^{A,i}, S_t^{H,i}) : \Omega^t \to R^3.$$

The expected utility with the probability measure $\mathcal{P}$ is defined as

$$E_0^{\mathcal{P}} \sum_{t=0}^{\infty} \delta^t \frac{(C_t^i)^{1-\gamma}}{1-\gamma} \equiv \int_{\Omega} \sum_{t=0}^{\infty} \delta^t \frac{C_t^i(\omega^t)^{1-\gamma}}{1-\gamma} d\mathcal{P}(\omega).$$

4.3 Optimality Conditions

Since the objective function is concave and the feasible set is convex, the agent's optimal plan is characterized by the first order conditions

$$(C_t^i)^{-\gamma} P_t^j = \delta E_t^{\mathcal{P}}((C_{t+1}^i)^{-\gamma}(P_{t+1}^j + D_{t+1})) \quad j \in \{A, H\} \tag{6}$$

Using standard arguments, the RE solution is

$$P_t^{H.RE} = P_t^{A.RE} = \frac{\delta a^{1-\gamma}\rho_\epsilon}{1-\delta a^{1-\gamma}\rho_\epsilon} D_t \tag{7}$$

where $\rho_\epsilon = E[(\epsilon_{t+1}^c)^{1-\gamma}\epsilon_{t+1}^d] = e^{\gamma(1+\gamma)\frac{s_\epsilon^2}{2}} e^{-\gamma\rho_{c,d}s_c s_d}$. Prices are governed by the dividends and prices are equalized.

We now characterize the equilibrium outcome under learning. Following AMN, under an assumption that stocks represent a very small part of total income (see Appendix A.3), and denoting β_t^j the subjective expectation for the risk-adjusted stock price growth

$$\beta_t^j \equiv E_t^{\mathcal{P}}[(\frac{C_{t+1}}{C_t})^{-\gamma}(\frac{P_{t+1}^j}{P_t^j})] \quad j \in \{A, H\}. \tag{8}$$

The definitions of β_t^j together with the first-order conditions (6) give rise to the asset pricing equations

$$P_t^j = \frac{\delta a^{1-\gamma}\rho_\epsilon}{1-\delta\beta_t^j} D_t \quad j \in \{A, H\}. \tag{9}$$

We see from this equation that this model can generate price differences if $\beta_t^A \neq \beta_t^H$, even if the two stocks share the same dividends D_t. Despite knowing that fundamentals of the two shares are identical, the agent is not informed about the price mapping and can still have different beliefs about their future price growth due, for example, to the lack of common knowledge about other's preferences and beliefs. When an agent has different perspectives for future capital gains of the two shares, the different expectations feed into different current realized stock prices. This is a quite natural assumption in light of the fact that actual capital gains have differed across these markets (before the connection) and that the two shares are distinct. Prices will actually differ in equilibrium just because investors expect A- and H- prices to differ, even when investors are (internally) rational. Given the different expected capital gains across the two types of shares, current prices of A-share and H-share need to differ in a way such that the expected (risk-adjusted) dividend yields achieve this equalization as the expected risk-adjusted returns need to be equalized

across markets.

Generating different prices hinges on $\beta_t^A \neq \beta_t^H$, we now turn to showing a system of beliefs about prices that justifies this.

4.4 Belief-Updating Rule

This section fully specifies the subjective probability distribution for prices in $\mathcal{P}$, and the optimal belief-updating rule for the subjective beliefs β_t^j. We start with the popular assumption in the adaptive learning/internal rationality literature, namely that investors believe that the risk-adjusted stock price growth in both A- and H-share markets can be modeled as the sum of a persistent and a transitory component

$$\begin{aligned} \left(\frac{C_t}{C_{t-1}}\right)^{-\gamma} \frac{P_t^j}{P_{t-1}^j} &= b_t^j + \epsilon_t^j,\ \epsilon_t^j \sim iiN(0, \sigma_{\epsilon,j}^2) \quad j \in \{A, H\} \\ b_t^j &= b_{t-1}^j + \xi_t^j,\ \xi_t^j \sim iiN(0, \sigma_{\xi,j}^2) \end{aligned} \tag{10}$$

where b_t^j are persistent components, ϵ_t^j are transitory components. For now we assume that all innovations are independent from each other, in section 7.1 below we consider the more reasonable case when agents believe that prices of the two shares are correlated.

One way to justify these beliefs is that they are compatible with RE: The beliefs (10) encompass the RE beliefs in (7) as a special case in which agents believe that $\sigma_{\xi^A}^2 = \sigma_{\xi^H}^2 = 0$, and $\beta_0^A = \beta_0^H = a^{1-\gamma}\rho_\epsilon$. Therefore, this gives a sense in which the beliefs of agents are "close" to RE, as long as we consider small values for $\sigma_{\xi^j}^2$ and β_0^j are close to $a^{1-\gamma}\rho_\epsilon$ (as we will).

Agents can observe the realizations of the risk-adjusted growth but not the persistent and transitory components separately. Forecasting the price growth by estimating the persistent components b_t^j engenders a filtering problem. As is well known, for normally distributed initial priors $b_0^j \sim N(\beta_0^j, \sigma_{0,j}^2)$ the optimal Kalman filter in steady state

$$\beta_t^j = \beta_{t-1}^j + \alpha^j \left[(\frac{C_{t-1}}{C_{t-2}})^{-\gamma} \frac{P_{t-1}^j}{P_{t-2}^j} - \beta_{t-1}^j \right] \qquad j \in \{A, H\} \tag{11}$$

where α^j are optimal (Kalman) gains which measures how quickly the agents' beliefs are updated with new information. As is well known α^j is increasing in the signal-noise ratio $\frac{\sigma_{\xi^l}}{\sigma_{\epsilon^j}}$. As below we estimate the α's to be small this will be compatible with small $\sigma^2_{\xi^j}$ and, therefore, a small deviation from RE.

The learning schemes in the equation (11), as well as the pricing equation (9) can generate rich stock-price dynamics arising from a self-referential feedback channel between the stock price P_t^j and the subjective beliefs β_t^j. According to equation (9), a high (low) β_t^j will lead to a high (low) realized stock price in the $j-$market. This will reinforce the subjective beliefs, inducing an even higher (lower) β_{t+1}^j through the equation (11), thus leading to a higher (lower) $j-$stock price and so on. The difference between β_t^A and β_t^H can be generated from a small difference in initial beliefs, or in the Kalman gains α^A and α^H, or in the addition of expectational shocks see section 6.2 below. Due to this self-referential property of the model, a difference in beliefs is able to generate persistent price differences between A-shares and H-shares.[35]

4.5 Quantitative Performance

This subsection presents the simulation outcomes for the model where agents learn about prices. The model is calibrated at a weekly frequency.[36] We set relative risk-aversion γ=5, then calibrate the mean and standard deviation of the dividend growth a, $\sigma_{\Delta D/D}$, the standard deviation of the consumption growth $\sigma_{\Delta C/C}$, and the correlation between consumption growth and dividend growth $\rho_{c,d}$ using the data on the Shanghai stock market and on Chinese consumption per capita. The data source is described in the appendix A.6. In particular, we follow the method as in Campbell (2003) to construct the quarterly dividend. Then, we use quarterly dividend and consumption data to calibrate weekly parameters following Campbell and Cochrane (1999).[37] We also calibrate

[35] As is standard in this literature agents' subjective beliefs β_t^j are truncated by a projection facility, this is detailed in Appendix A.5.

[36] We use weekly frequency because we have the weekly price; the data sample will be small if we aggregate it to monthly or quarterly frequency. And high frequency stock market data are widely used in the literature, for example Luca and Moench (2015) documented that there is a pre-FOMC announcement excess return using daily and hourly data.

[37] Campbell and Cochrane (1999) use annual data to calibrate monthly parameters. In our calibration, we divide the mean of quarterly net growth of dividend (and of consumption) by 13 to get the weekly mean of net growth. And we obtain the weekly standard deviation of dividend (and of consumption) growth by dividing the standard deviation of quarterly dividend growth by $\sqrt{13}$.

Parameters	Value
γ	5
$\sigma_{\Delta D/D}$	0.0204
$\sigma_{\Delta C/C}$	0.0025
a	1.0014
$\rho_{c,d}$	-0.03
δ	0.999
α^A	0.0030
α^H	0.0015

Table 2: Parameters Values for Learning Model

δ to match the 4% annual interest rate, that is, we obtain the weekly interest rate from the no-arbitrage condition and then calibrate δ according to the weekly interest rate. We set subjective expectation β_0^A and β_0^H in period 0 at the RE belief. We also set β_1^H in period 0 at the RE belief but set β_1^A slightly larger than β_1^H such that we get 1.6 percent premium as is observed in the data on connection in November 2014.[38]

We use the method of simulated moments (MSM) to estimate α^A and α^H, matching the mean, standard deviation, and persistence of the AH premium. Table 2 contains the parameter values for the model. The estimation results show that $\alpha^A > \alpha^H$, which implies agents perceive a larger signal-noise ratio for the A-share. Intuitively, if agents believe that the signal-noise ratio for A-share prices than H-share prices (the volatility of trend component contribute relatively more to the fluctuation of A-price than to that of H-price), agents will tend to update their beliefs faster in the face of changes in A-share prices. [39]

We run a Monte-Carlo simulation of the learning model for $10,000$ samples, with 100 periods for each sample to match the sample period of about 2 years. Figure 4 depicts one simulation of the A-share price P_t^A and H-share price P_t^H. A higher learning speed α^A for A-share prices leads P_t^A to rise and fall at higher speed relative to P_t^H while the two price dynamics retain the similar shape. This shows that the learning model has potential to produce a large and persistent price difference.

Table 3 contains the simulation results of the targeted moments. Column 2 shows the moments

[38] Alternatively, we could use the historical data to back out β_1^A and β_1^H given that agents' belief follows (11).

[39] Suppose that processes (10) are the date generating process (in fact, as is in subsection 4.6, (10) are compatible with actual data), the actual data of P_t^A and P_t^H also support this inequality by using the maximum likelihood estimation (MLE) method to estimate the signal-noise ratios.

Targeted Moments	**Data**	**Model** (95% interval)
$E(\frac{P_t^A}{P_t^H} * 100)$	130.71	[97.23 135.58]
$\sigma(\frac{P_t^A}{P_t^H} * 100)$	8.97	[2.99 22.60]
$\rho(\frac{P_t^A}{P_t^H})$	0.83	[0.89 0.99]

Table 3: Model Simulated Moments: Target Moments

Untargeted Moments	**Data**	**Model** (95% interval)
$corr(P_t^A, P_t^H)$	0.91	[0.76 0.99]
$corr(P_t^A - P_t^H, P_t^H)$	0.33	[0.12 0.96]
$\sigma(P_t^A)/\sigma(P_t^H)$	1.35	[1.21 2.79]

Table 4: Untargeted Simulated Moments

from the AH premium data while Column 3 reports the 95% intervals of the model's simulated moments. We find that the mean and standard deviation of the data are located within the intervals, although the model generates a slightly more persistent AH premium than the data.[40] Therefore, the model simulated moments are broadly consistent with the data in the sense that the learning model produces similar moments as in the data.

The fact that the A-share price adjusts more quickly than H-price is displayed graphically in Figure 3; a formal statistic capturing this behavior is a positive correlation of the price gap $(P_t^A - P_t^H)$ and H-price, and a relatively high variance of the A-share: a positive correlation $corr(P_t^A - P_t^H, P_t^H)$ shows that price of A tend to rise (fall) more when price of H rises (falls), while a higher variance for A-shares shows that the A-share is the one that is more volatile. The empirical and simulated results in Table 4 confirm these results. Even though we don't target those moments along with the correlation of A-price and H-price *i.e.* $corr(P_t^A, P_t^H)$, the model is quite good in terms of generating these untargeted moments.

Observing that our learning model generates similar targeted and untargeted moments as in the data, while the same model but with RE discussed in subsection 3.2 has difficulty in producing the data-like AH premium, our quantitative results confirm the key role of learning about prices in equity pricing. Appendix A.4 extends the learning model to cover both the segmentation and connection contexts, from 2006 to 2016.

[40] By introducing the preference shock to δ, the persistence could be reduced.

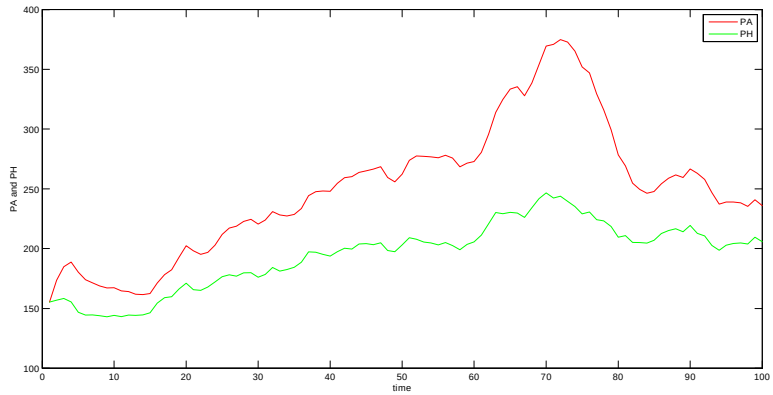


Figure 4: Simulated Stock Prices for A-Share and H-Share

4.6 Testing for the Rationality of Price Expectations

The idea of Internal Rationality is that even though agents do not know the pricing function they entertain beliefs that are not big deviations of rational expectations, in the sense that agents do not make large, easily detected mistakes. we should make sure that the postulated belief system is compatible with the actual data. As we explained in subsection 4.4, by choosing small $\sigma^2_{\xi^j}$ and initial conditions β^j_0 close to $a^{1-\gamma}\rho_\epsilon$ gives a precise sense of "closeness" of beliefs to RE beliefs. In this subsection, we examine what would happen if agents compare their beliefs to the actual data, and we explore whether agents who believe in the model (10) will be able to reject their beliefs with the actual data.

For this purpose, we use a set of testable restrictions implied by the agents' belief system developed by Adam, Marcet and Nicolini (2016) section 6 to test the belief system against the actual data.[41] Denote $x_t = (e_t, D_t/D_{t-1}, C_t/C_{t-1})$, where $e_t \equiv \Delta(\frac{C_t}{C_{t-1}})^{-\gamma}\frac{P_t}{P_{t-1}}$, with Δ representing the first difference operator. These restrictions are listed as follows:

Restriction 1: $E(x_{t-i}e_t) = 0$ for all $i \geqslant 2$,

Restriction 2: $E((\frac{D_t}{D_{t-1}} + \frac{D_{t-1}}{D_{t-2}}, \frac{C_t}{C_{t-1}} + \frac{C_{t-1}}{C_{t-2}})e_t) = 0$,

Restriction 3: $b'_{DC}\sum_{DC} b_{DC} + E(e_t e_{t-1}) < 0$,

Restriction 4: $E(e_t) = 0$,

[41] Klaus, Marcet, and Nicolini (2016) show that the postulated belief system is consistent with the model implied simulated data.

	Test Statistics A (H)	**5% Critical Value**
Restriction 1 using $\frac{D_t}{D_{t-i-1}}$	2.81 (0.76)	9.48
Restriction 1 using $\frac{C_t}{C_{t-i-1}}$	4.02 (4.77)	9.48
Restriction 1 using $\Delta(\frac{C_{t-i}}{C_{t-i-1}})^{-\gamma}\frac{P_{t-i}}{P_{t-i-1}}$	2.13 (2.55)	9.48
Restriction 2	0.04 (0.15)	5.99
Restriction 3	-3.55 (-3.60)	1.64
Restriction 4	0.002 (0.001)	3.84

Table 5: Testing Subjective Beliefs against Actual Data

where $\sum_{DC} \equiv var(\frac{D_t}{D_{t-1}}, \frac{C_t}{C_{t-1}})$ and $b_{DC} \equiv \sum_{DC}^{-1} E((\frac{D_t}{D_{t-1}}, \frac{C_t}{C_{t-1}})' e_t)$.

These four restrictions are necessary and sufficient conditions for the agents' beliefs to be compatible with $\{x_t\}$ in terms of second-order moments.[42] Under standard assumptions, any process satisfying these testable restrictions can - in terms of its autocovariance function - be generated by the postulated system of beliefs in the equation (10). Table 3 reports the statistics testing Restrictions 1-4 with the actual data.

The 5% critical values of the test statistics are reported in the last column of Table 3. For restriction 1, we compute the risk-adjusted consumption growth with $\gamma = 5$. The table shows that the test statistics in all cases are below their critical values and often by a wide margin. It then follows that agents can find the observed financial data to be compatible with their belief system. If they did a test against their belief system, they would not reject their belief. The belief system is one that agents could have reasonably held.

5 The Convergence Trading Strategy

A typical convergence trading strategy by investors is to hold long positions in an asset they consider undervalued and short positions in the other one with same or similar fundamentals they consider overvalued in the expectation that the prices of two assets will converge in the future. In our case, the convergence trading strategy is to short sell A-share while long buy H-share. A famous example is the convergence trading of the hedge fund Long-Term Capital Management

[42] Adam, Marcet and Nicolini (2016) prove the conditions are necessary and sufficient.

(LTCM).[43] As we mention in the introduction, it was reported that some arbitragers suffered a big loss and quickly dropped the convergence trading strategy. It is natural and interesting to ask what would happen in the model if some agents implement the convergence trading strategy after observing the large premium caused by internally rational investors with subjective price expectations.

The convergence trading is motivated by the convergence traders (arbitragers)' expected profits from the convergence of prices. Actually, according to the results in Chapter 7 of Evans and Honkapoha (2001), agents' price expectations $\beta_t^{A(H)}$ in section 4 will converge in distribution as

$$\lim_{t\to\infty} \beta_t^j \sim N(\beta^{RE}, \alpha^j Z^j), j = A, H$$

where Z^j is a number given in their Theorem 7.9. So the difference in capital gain expectations *i.e.* $\beta_t^A - \beta_t^H$ will converge in distribution with a mean of 0. The initialization of Shanghai-Hong Kong stock connect can promote price convergence in the long run.

However, the agents who learn about prices will not take the convergence trading strategy because their behavior is optimal given their higher expected capital gain about A- as opposed to H-share. Therefore we need to introduce other investors with superior information (or with rational expectation) into the model to explore how much profits they could earn by short selling A-shares and long buying H-shares with the income from selling A-shares.

Edwards (1999) and Shleifer and Vishny (1997) among others show that the limits of arbitrage are very restrictive in U.S. and Europe. Naked short-selling is not allowed, and the arbitragers are very likely to confront the funding liquidity issue as the external funding to arbitragers is usually performance-based. This is even more true for the arbitrage in China.[44] The maximum term of short selling in China is 1 year according to the stipulated regulation. Convergence traders are

[43] Edwards (1999) documented that LTCM expected the convergence of bond yields in emerging market countries and the US. Hence, they bought bonds from emerging markets and short-sold US government bonds in 1997. At the same time, according to Wikipedia, LTCM also bet that share prices of the dual-listed company Royal Dutch Shell would converge because of the same fundamental values. However, the spread of bond yields and price differences of stocks widened unexpectedly, which made LTCM incur a large loss from this strategy and led to the near-collapse of LTCM.

[44] The regulations of short selling including maximum term and guarantee ratio can be found in the official website of SSE. (http://www.sse.com.cn/services/tradingservice/margin/rules/c/c_20161219_4216850.shtml)

required to have at least 50% of the market value of short-selling assets in their accounts as a security deposit. And the guarantee ratio defined below has to be larger than 130%. They will receive a margin call and are forced to remargin (to provide additional cash or collaterals) when the guarantee ratio is below 130%. Thereby, only a small number of convergence traders with measure of zero are considered.[45]

Given that the A-share price is higher than H-share, the guarantee ratio gr_t for short selling A-share is defined as

$$gr_t \equiv \frac{0.5 * P_T^A + P_t^H * \frac{P_T^A}{P_T^H}}{P_t^A} \tag{12}$$

where T is period starting to implement convergence trading, and $P_t^H * \frac{P_T^A}{P_T^H}$ is the market value of H-shares bought through selling 1 unit of an A-share. It is a ratio of the sum of the security deposit and the market value of the asset to the market value of debt. Involuntary liquidation could occur due to a lack of enough capital to remargin receiving the margin calls. Liquidating part of their position at a highly unfavorable moment can result in a great loss. We do not model convergence traders' decision making about remargining except to point out the odds of receiving margin calls is non-negligible.

We consider a scenario where the convergence traders observing the AH premium start to implement their strategy standing in October 2016 which is the end of our data sample. We substitute actual A-prices and H-prices into the belief updating scheme (11) to recover the price expectations in October 2016 as the initial beliefs.[46]We run the Monte-Carlo simulation of 10,000 paths, with each path covering 52 periods (about 1 year).

Our simulation shows that convergence traders are likely to experience marginal calls as A-prices and H-prices are very likely to diverge in the short-run caused by agents learning about prices. The convergence traders can pay back the A-share at any time after period T but before period $T+52$. The probability of receiving a margin call *i.e.* $gr_t < 130\%$ in each period is obtained

[45]As we will discuss in the conclusion, it is interesting to explore whether introducing some more arbitragers into the model will promote the convergence or not. It might not be as obvious as it seems, as many papers such as Blume and Easly (2006), Cogley and Sargent (2009) and Dumas, Kurshev and Uppal (2009) point out that it could be that the agents with more information will be driven out of the market in the incomplete market or that it takes a very long time for the learning agents to be driven out of the market.

[46]We keep the consumption growth at its mean value because weekly consumption data doesn't exist.

	Mean	**Std**	Pr $(\boldsymbol{\pi}_t < \mathbf{0})$
3m	0.384	7.583	0.404
6m	-0.264	12.128	0.406
9m	-0.839	16.807	0.389
1y	-1.523	20.721	0.389

Table 6: The Statistics of Profits from Convergence Trading Strategy

by calculating the fraction of paths with $gr_t < 130\%$ in each following period. The maximal probability over each period can reach as high as 13.4%, which means that the convergence traders could receive a margin call with a probability of 13.4% in some periods. At the same time, we get the distribution of convergence trading profits π_t by paying back the A-share in period t in the scenario absent the requirement of security deposit:

$$\pi_t \equiv P^H_{T+t} * P^A_T / P^H_T - P^A_{T+t}.$$

We show the distribution of profits for $t = 13, 26, 39$ and 52, corresponding to 3 months, 6 months, 9 months, and 1 year. Table 6 display the means, standard deviations of π_t and probabilities of negative profits *i.e.* $\pi_t < 0$. As apparent in Table 6, the mean of π_t is a small number, either positive or negative; the standard deviations (Std) are large implying convergence trading is very risky; the probability of negative profits are high. Figure **??** plots the distributions of 1-year-convergence trading profit, the left-fat-tail of which indicates a high probability of a great loss.[47] This would prevent a small number of investors from taking the convergence trading strategy.

6 Extensions

In this section, we discuss some extensions to the benchmark learning model in section 4.

[47] If we take the security deposit and margin calls into account, convergence trading would be riskier. This table gives the lower bound of the loss because we didn't take into account of the margin call when calculating the profits.

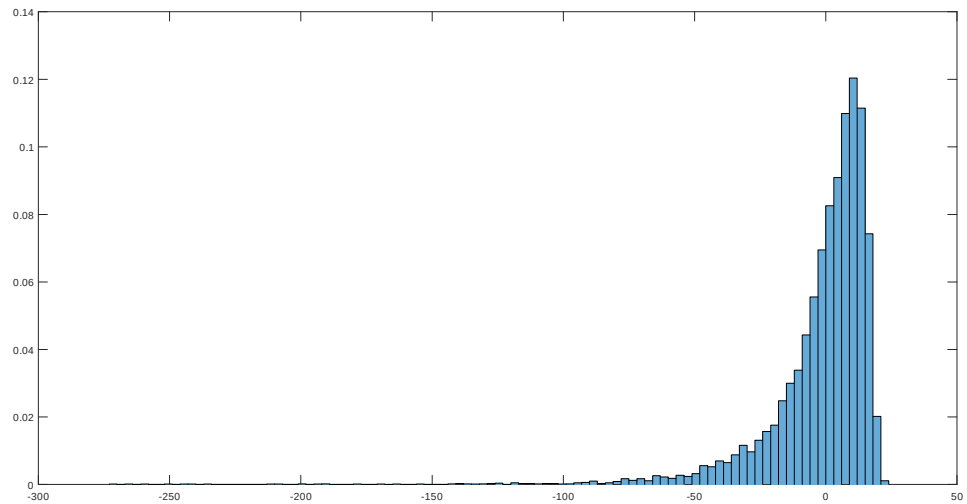


Figure 5: The Distribution of Profit by Implementing Convergence Trading Stragtegy (1 year)

6.1 Cross-Learning Scheme

Figure 3 displays that A-share and H-share prices are highly positively correlated. By observing this high correlation, agents probably think that the price of one class of shares can provide information for the other class of shares. To capture this feature, instead of modeling agents' belief system such that two prices are perceived to be independent of each other, it is natural to model it as the cross-learning scheme as follows:

$$\begin{bmatrix} (\frac{C_t}{C_{t-1}})^{-\gamma}\frac{P_t^A}{P_{t-1}^A} \\ (\frac{C_t}{C_{t-1}})^{-\gamma}\frac{P_t^H}{P_{t-1}^H} \end{bmatrix} = \begin{bmatrix} b_t^A \\ b_t^H \end{bmatrix} + \begin{bmatrix} \epsilon_t^A \\ \epsilon_t^H \end{bmatrix}$$

$$\begin{bmatrix} b_t^A \\ b_t^H \end{bmatrix} = \begin{bmatrix} b_{t-1}^A \\ b_{t-1}^H \end{bmatrix} + \begin{bmatrix} \xi_t^A \\ \xi_t^H \end{bmatrix}$$

where $(\epsilon_t^A \ \epsilon_t^H)' \sim N(0, R)$ and $(\xi_t^A, \xi_t^H)' \sim N(0, Q)$, R and Q could be non-diagonal variance-covariance matrices. The independent belief system is a special case of the cross-learning scheme

Moments	**Data**	**Model** (95% interval)
$E(\frac{P_t^A}{P_t^H} * 100)$	130.71	[96.12 136.40]
$\sigma(\frac{P_t^A}{P_t^H} * 100)$	8.97	[3.46 25.27]
$\rho(\frac{P_t^A}{P_t^H} * 100)$	0.83	[0.90 0.99]
$corr(P_t^A, P_t^H)$	0.91	[0.66 0.99]

Table 7: Model Simulated Moments

in which R and Q are diagonal matrices. Agents optimally update their beliefs according to:

$$\begin{bmatrix} \beta_t^A \\ \beta_t^H \end{bmatrix} = \begin{bmatrix} \beta_t^A \\ \beta_t^H \end{bmatrix} + \begin{bmatrix} \alpha^A & \alpha^C \\ \alpha^C & \alpha^H \end{bmatrix} \begin{bmatrix} (\frac{C_{t-1}}{C_{t-2}})^{-\gamma} \frac{P_{t-1}^A}{P_{t-2}^A} - \beta_{t-1}^A \\ (\frac{C_{t-1}}{C_{t-2}})^{-\gamma} \frac{P_{t-1}^H}{P_{t-2}^H} - \beta_{t-1}^H \end{bmatrix}$$

so that agents use not only the newly observed A-price (H-price) but also the H-price (A-price) to update their belief about future capital gain of A-shares (H-shares).[48]

The pricing equation is still (11). We use MSM again to estimate the parameters $[\alpha^A \ \alpha^H \ \alpha^C]$ to match the mean, standard deviation, persistence of AH premium as well as the correlation of A-share price and H-share price in the whole sample. The estimated parameters are $\alpha^A = 0.0032, \alpha^H = 0.0015, \alpha^C = 0.0001$. Note that α^C is smaller than α^A and α^H with one order of magnitude. The Monte-Carlo simulation results are in the Table 7. We can find that the 95% intervals of the simulated moments are very close to those in section 4. The simulated two prices are highly correlated.

6.2 Learning with Expectation Shocks

Expectation shocks are shown to be important for business cycle and asset prices in the literature, e.g. Bullard, Evans and Honkapohja (2008) and Milani (2011, 2017). They are identified as the exogenous component of expectations not accounted for by the learning model.[49] We incorporate

[48] Define X such that $X(X+R)^{-1}X = Q$. The Kalman gain vector $K \equiv \begin{bmatrix} \alpha^A & \alpha^C \\ \alpha^C & \alpha^H \end{bmatrix}$ is obtained by $K = X(X+R)^{-1}$.

[49] Bullard, Evans and Honkapohja (2008) call this exogenous component "judgment'.

expectation shocks into the learning model by modeling the belief updating scheme as follows

$$\beta_t^j = \beta_{t-1}^j + \alpha^j((\frac{C_{t-1}}{C_{t-2}})^{-\gamma}\frac{P_{t-1}^j}{P_{t-2}^j} - \beta_{t-1}^j) + \epsilon_t^{\beta^j} \quad j \in \{A, H\} \tag{13}$$

where $\epsilon_t^{\beta^j} \sim i.i.N(0, (\sigma^j)^2)$ are the expectation shocks. The difference in expectation shocks are able to give rise to the difference in β_t^A and β_t^H. The expectation shock can be regarded as the shock to agents' informations set (Adam, Marcet and Beutel 2017). Alternatively, expectation shocks can be understood as the small deviations from internal rationality – waves of optimism and pessimism. If agents are fully internally rational, their optimal belief would follow the belief updating scheme in section 4 strictly. However, agents' expectation might deviate from the optimal forecast: they can be either over-optimistic – by believing that future capital gains will be higher than those predicted by their learning model – or over-pessimistic. The initial higher expected capital gains about A-shares as opposed to H-shares $\beta_1^A > \beta_1^H$ could come from more optimistic expectation shock to A-share than to H-share which might be related to the widely spread optimistic media reports on the A-share market in 2014 as the result of the narrative of reform-based bull market.[50]

We simulate the model to understand the effects of expectation shocks on the AH premium and convergence trading. Instead of estimating parameter values of expectation shocks as in Milani (2011), we set the standard deviations σ^A and σ^H at 0.00005 and 0.00002, which are ten percentage of the steady-state standard deviations of β^A and β^H in section 4. Table 8 shows the model simulated moments. We find that expectation shocks lead to higher and more volatile price differences; the model simulated moments are still broadly compatible with data. A-share and H-share prices are more likely to diverge in the short-run. Hence the probability of receiving marginal calls ($gr_t < 130\%$) becomes larger and can reach as high as 21.4%. And as apparent in table 9, we see a more negative mean and larger standard deviation of convergence trading profit π_t as well as a higher probability of negative profit. This is also illustrated by a more-skewed left-fat-tailed distribution of profit as in Figure 6.

[50] See a discussion in Huang, Miao and Wang (2019).

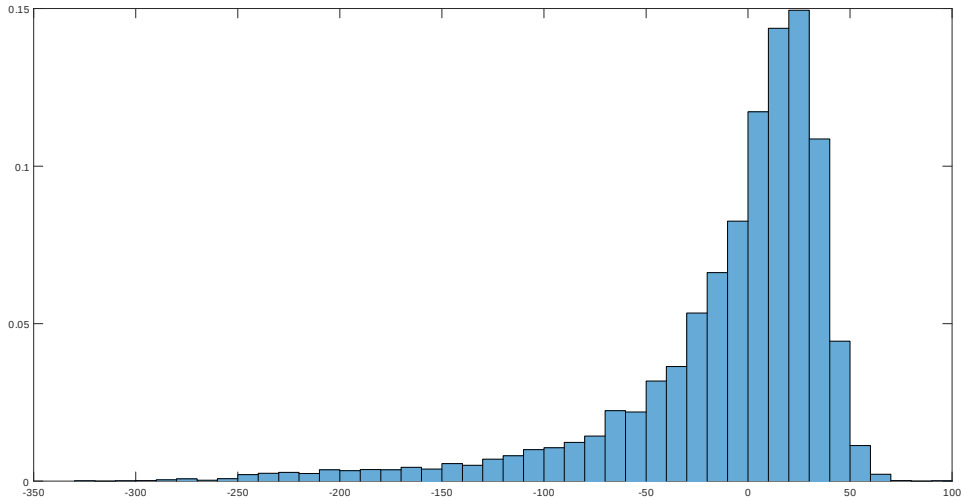


Figure 6: The Distribution of Profit by Implementing Convergence Trading Stragtegy (1 year)

Moments	**Data**	**Model** (95% interval)
$E(\frac{P_t^A}{P_t^H} * 100)$	130.71	[92.64 143.27]
$\sigma(\frac{P_t^A}{P_t^H} * 100)$	8.97	[3.96 28.12]
$\rho(\frac{P_t^A}{P_t^H} * 100)$	0.83	[0.90 0.99]

Table 8: Model Simulated Moments

	Mean	**Std**	$\Pr(\boldsymbol{\pi}_t < \mathbf{0})$
3m	-2.127	23.787	0.447
6m	-7.100	42.026	0.438
9m	-8.762	48.966	0.432
1y	-9.496	53.475	0.423

Table 9: The Statistics of Profits from Convergence Trading Strategy

7 Conclusion

This paper explains the AH premium. Since this is a large market we think of these results as a natural experiment to test asset pricing models. We show that many versions of Bayesian/RE asset equilibria where agents know the pricing function in connected markets fail to generate a data-like AH premium, including models with RE, agents that are heterogeneous in beliefs and preferences, transaction costs, dividend taxation, complete and incomplete markets. We also discuss informally some possible extensions.

A model of learning about prices explains the price differences of dual-listed companies in a very natural way. The reason is that small differences in beliefs about the stocks in different markets provide a "nearly" self-fulfilling prophecy that prices are different. Although agents do not know exactly the pricing function they hold beliefs that are close to the actual behavior of the model. Therefore, we think of AH premium as a natural experiment rejecting Bayesian/RE models in favor of learning about prices in terms of expectation formation for stock prices. Of course, the factors uncovered by Bayesian/RE models could explain part of the story but they seem not to be the main characters in the story. Heterogeneous beliefs and transaction costs may act as good supporting actors for this story in future research, but the main actor seems to be learning about prices.

When the internal rational agents learn about stock prices, just because investors expect A- and H- prices to differ these prices will actually differ in equilibrium. Agents make "mistakes" that would be undetectable statistically and, therefore, are not really "mistakes". We also show that convergence trading strategy is highly likely to suffer a great loss because the prices might diverge in the short run, so that even if someone understands this property of the model this will not cause learning about prices to generate incorrect investing strategies.

As we pointed out in section 3, further study in expectations of exchange rate behavior, rational bubble equilibria, and market liquidity should also be explored. As pointed out by Shleifer and Vishny (1997), when specialized arbitrageurs implementing the performance-based arbitrage only accounts for a small fraction of investors, such arbitrage is not effective in bringing stock prices to their fundamental values. However, as the classical literature shows (e.g. Fama 1965, Sharpe

1964), if the number of arbitrageurs is large, their collective actions should force stock prices to converge to their fundamental values. While we assume a zero measure of convergence traders as a shortcut in Section 6, it would be interesting to investigate whether a positive measure of arbitrageurs with rational expectation trading with learning agents will ride the wave or lean against the bubble. Gromb and Vayanos (2010) survey the limits of arbitrage and show that short-selling costs, leverage constraints and constraints on equity capital are the impediments to arbitrage. It also appears of interest to study the implications of lowering limits of arbitrage in this setting.

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8 Appendix

8.1 Basic Model Setting

Type i agents maximize their expected discounted utility subject to the budget constraint

$$\max_{\{C_t^i, S_t^{i,A}, S_t^{i,H}, B\}} E_0 \sum_{t=0}^{\infty} (\delta^i)^t u^i(C_t^i)$$

$$\begin{aligned} s.t. \qquad & S_t^{i,A} P_t^A + S_t^{i,H} P_t^H + C_t^i + B_t^i(D_h) Q_t(D_h) + B_t^i(D_l) Q_t(D_l) \\ = \quad & S_{t-1}^{i,A}(P_t^A + D_t^A) + S_{t-1}^{i,H}(P_t^H + D_t^H) + B_{t-1}^i(D_t) + Y_t^i. \end{aligned}$$

Type i agents consume C_t^i amount of goods, buy $S_t^{i,A}$ units of A-shares, $S_t^{i,H}$ units of H-shares and Arrow securities with price $Q_t(\cdot)$, and receive payment from the shares and Arrow securities bought in the previous period as well as constant endowment Y_t^i. The Arrow securities market-clearing conditions are

$$\mu^1 B_t^1(D_j) + \mu^2 B_t^2(D_j) = 0 \ \forall j = h, l$$

The supply of each share is assumed to be 1 so that the A- and H-share market-clearing condition is

$$\mu^1 S_t^{1,Z} + \mu^2 S_t^{2,Z} = 1 \ \ \forall Z = A, H.$$

The sum of consumption of the two agents is equal to the total dividend and endowment; hence the commodity goods market-clearing condition is

$$D_t + \mu^1 Y_t^1 + \mu^2 Y_t^2 = \mu^1 C_t^1 + \mu^2 C_t^2.$$

By Walras law, the goods market-clearing will be satisfied automatically.

8.2 Algorithms and Simulations for State-Contingent Bond Positions

8.2.1 Rational Expectation

We assume dividend payment is high in the first period, the two agents share the same subjective discount factor *i.e.* $\delta^1 = \delta^1$ and the endowment is constant *i.e.* $Y_t^1 = Y^1$ and $Y_t^2 = Y^2$.

Step 1:Draw N series of T periods each of dividends $\{\{D_{n,t}\}_{t=0}^T\}_{n=1}^N$ using a *i.i.d* random number generator with $D_{n,0} = D^h$. Guess a value for λ representing constant ratio of marginal utility in the complete market *i.e.* $\lambda = \frac{u'(C_t^1)}{u'(C_t^2)}$ and simulate for consumption $\{\{C_t^{1,n}, C_t^{2,n}\}_{t=0}^T\}_{n=1}^N$ using commodity goods market clearing condition. Solve for λ by iteration using the intertemporal budget constraint of type 1 agent as follows:

$$\frac{1}{N}\sum_{n=1}^{N}\sum_{t=0}^{T}\delta^t \frac{u'(C_t^1)}{u'(C_0^1)}(C_{t+j}^1 - D_{t+j} - Y^1) = B_{-1}^1$$

where B_{-1}^1 is the initial inherited holding of the bond.

Step 2: Let $Z = u'(c(D^l))(c(D^l) - D^l - Y^1)\pi + u'(c(D^h))(c(D^h) - D^h - Y^1)(1-\pi)$. We obtain $B_{t-1}^1(D^l)$ by

$$\begin{aligned} B_{t-1}^1(D^l) &= E_t \sum_{j=0}^{\infty} \delta^j \frac{u'(C_{t+j}^1)}{u'(C_t^1)}(C_{t+j}^1 - D_{t+j} - Y^1) \\ &= (C^1(D = D^l) - D^l) + \frac{\delta}{u'(C^1(D = D^l))(1-\delta)} Z \quad \forall t. \end{aligned}$$

	$\gamma^1 = 1$	$\gamma^1 = 2$	$\gamma^1 = 3$	$\gamma^1 = 4$
$C^1(D^h)$	1	0.8736	0.8045	0.7629
$C^1(D^h)$	0.5	0.5514	0.5717	0.5818
$B^1(D^l)$	0	0.1017	0.1416	0.1619
$P^A(D^h)$	100	112.64	119.55	123.71
$P^H(D^h)$	100	112.64	119.55	123.71
$P^A(D^l)$	50	44.97	42.99	41.99
$P^H(D^l)$	50	44.97	42.99	41.99

Table 10: Consumption and State-Contingent-Bond Holdings in Two States

$B^1_{t-1}(D^h) = B^1_{-1}$ because of the time-invariant policy functions and assumption of $D_0 = D^h$. The bond position of type 2 agents is opposite to that of type 1 agents by the market clearing condition for the bond market. Similarly, we can get the bond price.

In our simulation, we let $\delta = 0.99$, $\gamma^2 = 1, Y^1 = 0$, $Y^2 = 0$, $D^h = 1$, $D^l = 0.5$ and $prob(D_t = D^h) = prob(D_t = D^l) = 0.5$. We alter the risk aversion of type 1 agents while fixing that of type 2 agents to illustrate how risk aversion affects consumption and bond allocations.[51] When the two type agents have the same risk aversion $\gamma^1 = \gamma^2$, they share the same consumption without bond trading. As type 1 agents become more risk averse, they want a more smooth consumption profile by holding more contingent bond for D^l seeking to be compensated in the bad states. While varying γ^1 does affect the price of equities, it is consistent with P^A equal P^H. These results are shown in Table 10, where $x(D^h)$ means the value of x when $D_t = D^h$.

When agents have different subjective discount factors, say $\delta^1 < \delta^2$, then $\frac{u'(c^1_{t+1})}{u'(c^2_{t+1})} = \frac{\delta^2}{\delta^1}\frac{u'(c^1_t)}{u'(c^2_t)}$. In this case, the ratio of marginal utility is not constant and increases over time. Impatient type 1 agents will issue more and more state-contingent bonds, which requires them to consume less and less over time. In the limit, $\frac{u'(c^1_T)}{u'(c^2_T)} \to \infty$ as $T \to \infty$, this means that in the limit type 2 agents consume all the dividends $c^2_T \to y_T$ while type 1 agents consume nothing $c^1_T \to 0$. Given δ^2, the lower δ^1 is, the faster the speed of convergence to the limit is.

[51] With state-contingent bonds, holdings of A-shares and H-shares are not uniquely determined because the shares are 'redundant' assets. Yet, the value for consumption, bond positions and price are uniquely determined. In the simulation, we keep agents' shareholdings of the two assets fixed over time so that they don't trade stocks with each other.

8.2.2 Diverse Belief

Step 1: The first step 1 is to solve the ratio of marginal utility in the first period λ_0. It is the same as step 1 in the rational expectation case except that the change of measure by multiplying the ratio of subjective probability to objective probability should be taken into account.

Step 2 : Draw one long series of T_L periods of dividends using the random number generator. Simulate the time-varying marginal rate of substitution $\{\lambda_t\}$ follows:

$$\lambda_t = \alpha_{t-1}(D_t)\lambda_{t-1}$$

where $\alpha_{t-1}(D_t) = \frac{prob^2(D_t)}{prob^1(D_t)}$. Find the corresponding consumption $\{C_t^1, C_t^2\}_{t=0}^{T_L}$ by the market-clearing condition given $\{\lambda_t\}_{t=0}^{T_L}$. Calculate the present value of primary deficits denoted by $Dd_t^1 \equiv \sum_{j=0}^{T_l} \delta^{1,j} \frac{u'(C_{t+1}^1)}{u'(C_t^1)}(C_{t+j}^1 - D_{t+j})$ for agent 1 taking account of the change of measure, which can be solved backward assuming $Dd_{T_L}^1 = 0$.

Step 3: Run a regression of $\{Dd_t^1\}_{t=1}^{T_L-\bar{T}}$ on $\{D_t\}_{t=1}^{T_L-\bar{T}-}$ and $\{\lambda_{t-1}\}_{t=1}^{T_L-\bar{T}}$ with the last $\bar{T}$ periods omitted.[52] The time-varying bond positions of type 1 agents are the fitting values of the regression. The bond holdings of type 2 agents are the opposite of those of type 1 agents.

We use this method to simulate the model with diverse beliefs. The calibration for rational expectation case still applies with $prob^1(D_t = D^h) = 0.51$, $prob^1(D_t = D^h) = 0.54$ and $\gamma^1 = \gamma^2 = 1$.Type 1 agents are more accurate in terms of the distance of subjective probability from the true probability. As apparent in Figure 7 which depicts one simulation of the economy, they accumulate assets and consume more goods over time while type 2 agents behave in the opposite way. In the long run/limit, type 1 agents will consume the total resources while type 2 agents get nothing. It is the relative correctness of the perceived beliefs that drives this bond-trading pattern rather than their degree of optimism. While type 2 agents are more optimistic than type 1 agents, it is the type 2 agents who want to buy more bonds. However, although the two agents have heterogeneous perspectives about the economic fundamentals, the prices of the two shares are identical for all states.

[52] In this case, the state-contingent-bond positions are not only the function of $\{D_t\}$ but also a function of $\{\lambda_{t-1}\}$ *i.e.* $B_{t-1}^1(D^j) = E(Dd_t^1 | D_t = D^j, \lambda_{t-1})$.

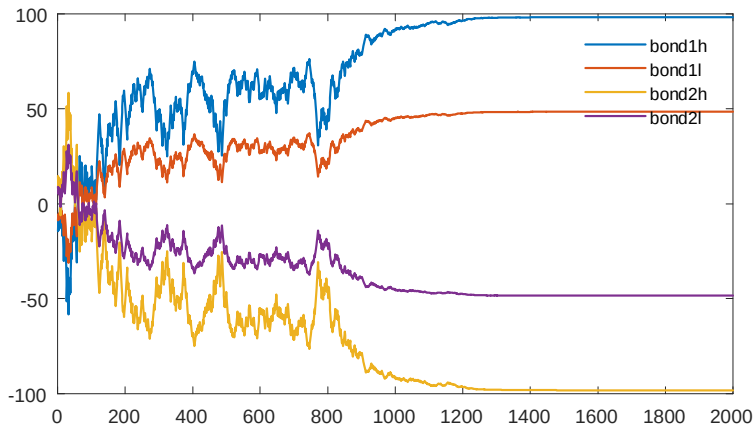

Figure 7: Agents' State-Contingent-Bond Positions

8.3 An Assumption for Learning

According to the arguments described by Adam, Marcet and Nicolini (2016), without strict rational expectations we may obtain $E^{\mathcal{P}}[C^i_{t+1}] \neq E^{\mathcal{P}}_t[C_{t+1}]$, even if $C^i_{t+1} = C_{t+1}$ holds ex-post in the equilibrium. We invoke similar approximations, as follows:

$$E^{\mathcal{P}}_t[(\frac{C^i_{t+1}}{C^i_t})^{-\gamma}(P^A_{t+1} + D_{t+1})] \simeq E^{\mathcal{P}}_t[(\frac{C_{t+1}}{C_t})^{-\gamma}(P^A_{t+1} + D_{t+1})] \tag{14}$$

$$E^{\mathcal{P}}_t[(\frac{C^i_{t+1}}{C^i_t})^{-\gamma}(P^H_{t+1} + D_{t+1})] \simeq E^{\mathcal{P}}_t[(\frac{C_{t+1}}{C_t})^{-\gamma}(P^H_{t+1} + D_{t+1})]. \tag{15}$$

The following assumption as in Adam, Marcet and Nicolini (2016) provides sufficient conditions for this to be the case:

Assumption We assume that Y_t is sufficiently large and that $E^{\mathcal{P}}_t P^{A(H)}_{t+1}/D_t < \overline{M}$ for some $\overline{M} < \infty$ so that, given finite asset bounds $\underline{S}$ and $\overline{S}$, the approximations (14) and (15) hold with sufficient accuracy.

8.4 A Learning Model Covering Segmentation and Connection Episodes

Figure 8 shows the historical changes of the AH premium index from 2006 to 2016. We see that not surprisingly, the two prices are not equal with a large trading premium for A-shares before the two markets connected. We will show that a modified internal rationality learning model can

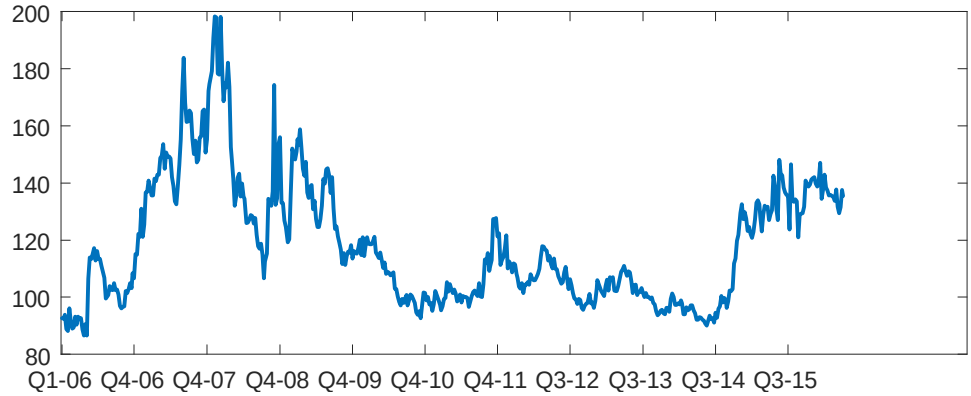

Figure 8: The Historical Changes in AH Premium Index from 2006 to 2016

also match the AH premium index both before and after the connection program, even though the focus of our paper is the price differences in connected markets and some other theories discussed in the literature review can explain the price differences when the two markets are segmented.

We extend the benchmark learning model to cover both segmentation and connection periods. To focus on the learning mechanism, we assume the dividend and consumption processes are the same across agents follwing the ones in section 5. The assumption in the appendix A.3 is still used. We only present how this model differs from the one in section 5.

The maximization problem for type 1 agents representing mainland investors is

$$\begin{aligned} & \text{Max}_{\{C_t^1, S_t^{A,1}, S_t^{H,1}\}} \; E_0^{\mathcal{P}^1} \sum_{t=0}^{\infty} \delta^t \frac{(C_t^1)^{1-\gamma}}{1-\gamma} \\ s.t. C_t^1 + P_t^A S_t^{A,1} &= (P_t^A + D_t) S_{t-1}^{A,1} + Y_t^1 \text{ if } t \leq N \\ C_t^1 + P_t^A S_t^{A,1} + P_t^H S_t^{H,1} &= (P_t^A + D_t) S_{t-1}^{A,1} + (P_t^H + D_t) S_{t-1}^{H,1} + Y_t^1 \text{ if } t \geq N \end{aligned}$$

$$0 \leq S_t^{A,1} \;\&\; 0 \leq S_t^{H,1}$$

where N represents the period when the two markets connect. And the maximization problem for

Moments	**Data**	**Model** (95% interval)
$E(\frac{P_t^A}{P_t^H} * 100)$	118.13	[97.44 121.48]
$\sigma(\frac{P_t^A}{P_t^H} * 100)$	21.79	[5.77 25.88]
$\rho(\frac{P_t^A}{P_t^H} * 100)$	0.97	[0.97 0.99]

Table 11: Model Simulated Moments

type 2 agents representing Hong Kong and foreign investors is

$$\begin{aligned} & \text{Max}_{\{C_t^2, S_t^{A,2}, S_t^{H,2}\}} \; E_0^{\mathcal{P}^2} \sum_{t=0}^{\infty} \delta^t \frac{(C_t^2)^{1-\gamma}}{1-\gamma} \\ s.t. C_t^2 + P_t^H S_t^{H,2} &= (P_t^H + D_t) S_{t-1}^{H,2} + Y_t^2 \text{ if } t \leq N \\ C_t^2 + P_t^A S_t^{A,2} + P_t^H S_t^{H,2} &= (P_t^A + D_t) S_{t-1}^{A,2} + (P_t^H + D_t) S_{t-1}^{H,2} + Y_t^2 \text{ if } t \geq N \end{aligned}$$

$$0 \leq S_t^{A,2} \;\&\; 0 \leq S_t^{H,2}$$

The pricing equations according to Adam and Marcet (2011) are:

$$P_t^A = \{ \begin{array}{c} \frac{\delta (a)^{1-\gamma} \rho_\epsilon}{1-\delta \beta_t^{1,A}} D_t \text{ if } t \leq N \\ \max_{i \in 1,2} \frac{\delta (a)^{1-\gamma} \rho_\epsilon}{1-\delta \beta_t^{i,A}} D_t \text{ if } t > N \end{array}$$

$$P_t^H = \{ \begin{array}{c} \frac{\delta (a)^{1-\gamma} \rho_\epsilon}{1-\delta \beta_t^{2,H}} D_t \text{ if } t \leq N \\ \max_{i \in 1,2} \frac{\delta (a)^{1-\gamma} \rho_\epsilon}{1-\delta \beta_t^{i,H}} D_t \text{ if } t > N \end{array}$$

And the belief updating scheme is

$$\beta_t^{i,j} = \beta_{t-1}^{i,j} + \alpha^j ((\frac{C_{t-1}^i}{C_{t-2}^i})^{-\gamma} \frac{P_{t-1}^j}{P_{t-2}^j} - \beta_{t-1}^j) \;\; i \in \{1,2\}, \; j \in \{A, H\}$$

We use the same parameter values as in section 5. The simulated outcomes are shown in Table 11. The 95% interval of the model's simulated moments contain the moments of realized data. The internal rationality learning mechanism can explain both the segmented and connected periods.

8.5 Differentiable Projection Facility

The function ω for the differentiable projection facility is:

$$\omega(\beta) = \left\{ \begin{array}{cc} \beta & \text{if } x \leq \beta^L \\ \beta^L + \frac{\beta - \beta^L}{\beta + \beta^U - 2\beta^L}(\beta^U - \beta^L) & \text{if } \beta^L < x \leq \beta^U \end{array} \right\}$$

In our numerical exercise, we choose β^U such that the implied price-dividend ratio never exceeds $U^{PD} = 600$ and set $\beta^L = \delta^{-1} - 2(\delta^{-1} - \beta^U)$.

8.6 Data Sources

The data are China financial and macroeconomic data. Financial data including Hang Seng China AH Premium Index, Hang Seng China A index, Hang Seng China H index, Shanghai Composite Index and dividend yield are downloaded from the Wind Financial Database (http://www.wind.com.cn). The sample period of Hang Seng China AH Premium, A and H Index is from Jan 2006 to Oct 2016. The daily (monthly) price series has been transformed into a weekly series by extracting the value of the last trading day of the corresponding week (month). The dividend yield is a moving average of the dividends in the previous 11 months and the current month divided by the Shanghai Composite Index. The monthly dividends on the index portfolio are obtained from the monthly dividend yield under the assumption that dividends have been approximately constant during the last 12 months. We have quarterly dividends by adding up the corresponding monthly series. Following Campbell (2003), we deseasonalize dividends by taking averages of the actual dividend payments over the current and preceding three quarters. The sample period of dividend yield is from 1995 Q1 to 2015 Q4.

Macroeconomic data including consumption, 1-year deposit interest rate and CPI in quarterly frequency are downloaded from Fed Atlanta (https://www.frbatlanta.org/cqer/research/china-macroeconomy.aspx?panel=1), which are used in Chang et al. (2016). The sample period is from 1995 Q1 to 2015 Q4. To obtain real values, nominal variables are deflated using China CPI.

Transaction costs and dividend taxes can be found in the official websites of regulatory authorities and stock exchanges. Transaction costs in SSE are in "http://www.sse.com.cn/services /trad-

ingservice/charge/ssecharge/", transaction cost in HKSE in "https://www.hkex.com.hk/Services/Rules-and-Forms-and-Fees /Fees/Securities-(Hong-Kong)/Trading/Transaction?sc_lang=en". Dividend taxes in connect program can be found in "http://news.stcn.com/2014/1118/11856372.shtml", dividend taxes in SSE for mainland investors in "http://www.csrc.gov.cn/shenzhen/xxfw /tzzsyd/zqtz/201307/t20130723_2 dividend taxes in HKSE for Hong Kong and international investors in "https://www.hkex.com.hk/-/media/hkex-market/services/circulars-and-notices /participant-and-members-circulars/hkscc/2009 /ce_127_2009".