# Information Frictions and the Paradox of Price Flexibility*

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**Abstract**

The introduction of digital price tags may facilitate price adjustments and reduce the degree of nominal rigidity in the economy. Is this welfare improving? We address this question in a New Keynesian model, emphasizing the role of information frictions and dispersed beliefs. In such a model, in contrast to a model with perfect information, price dispersion within re-optimizing firms arises. The welfare losses associated with the latter are amplified with a more flexible price. In a static model, we derive analytically that consumers' welfare losses are increasing in the price flexibility — the paradox of price flexibility. The results carry over to a dynamic quantitative model.

Keywords: Welfare, Price flexibility, Dispersed belief, Information Frictions, DSGE
JEL Classification:

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# 1 Introduction

Electronic shelf labels (ESL) permits retailers to set price digitally without any costs that would otherwise occur using paper price tags. Over the past decade, we have witnessed an expansion in the usage of digital price tags thanks to the growing affordability of ESL.[1] The introduction of digital price tags may facilitate price adjustment and reduce the degree of nominal rigidity in the economy. Is such a technological progress welfare improving?

We address this research question in a NK model with both nominal rigidity and dispersed information. Both frictions are shown in the literature to be relevant empirically.[2] We highlight a new channel — *dispersed beliefs channel* that is relevant to understand the welfare consequence of a reduction in nominal rigidity.

In a standard NK model, dispersions in prices and quantities arise due to price stickiness. Nominal rigidity such as staggered prices *à la* Calvo (1983) creates dispersion in prices across those firms because of staggered price setting. We denote this as the *Calvo channel*. Such dispersions are inefficient because goods matter for households' utility symmetrically and production technology are identical. Dispersed information that gives birth to imperfect common knowledge or dispersed beliefs creates another channel through which price dispersion arises. Firms have different assessments about the state of the economy due to information frictions. Therefore, in contrast to a standard model with perfect information, price dispersion emerges among those firms who can reset prices. We denote this as the *dispersed beliefs channel*.

To fully understand the welfare consequence of a change in price flexibility, we derive the

[1] See the report by Global Market Insights: https://www.globenewswire.com/news-release/2018/09/18/1572161/0/en/Electronic-Shelf-Label-ESL-Market-to-hit-1bn-by-2024-Global-Market-Insights-Inc.html

[2] See for an example Nakamura and Steinsson (2008) for empirical evidence on nominal rigidity from the micro level prices data. Empirical evidences supporting the presence of information frictions and dispersed beliefs are abundant, see for examples Coibion and Gorodnichenko (2012, 2015), Andrade *et al.* (2016) and Coibion *et al.* (2018) for a recent survey.

welfare loss function around the perfect information steady state and decompose it into three components. The first component, through the *Calvo channel*, is proportional to the price dispersion across all firms as if the newly price resetting firms are restricted to set the same average price. This component is a hump shape function of price rigidity. The second component, arising from the dispersed belief channel, is proportional to the price dispersion across the newly set prices. The associated welfare losses increase monotonically in the degree of price flexibility. In one extreme case, if the price is fully rigid, there is no dispersion in prices even if firms have different assessments about the state of the economy. In another extreme case when the prices are fully flexible, firms disagree with each other and would set the price based on their private information. Then the welfare losses associated with the price dispersion originating from the imperfect common knowledge would be maximized. The third component is proportional to the output gap volatility, caused by both nominal and information frictions, which is increasing in the price rigidity. The aggregate effect of an increased price flexibility on welfare is thus ambiguous.

In a static model, we derive analytically the conditions under which the dispersed beliefs channel dominates. Consequently, under those conditions an improved pricing technology that facilitates price adjustment may be associated with bigger welfare losses — the paradox of price flexibility. Two parameters are crucial to this finding. Not surprisingly, the first is the signal-to-noise ratio that characterizes the degree of information frictions, and thus the degree of disagreement among firms. The latter leads to a proportionally higher price dispersion among price resetting firms. The second is the parameter that characterizes the relative importance of price dispersion and output gap volatility in the welfare loss function. This is the case because the price dispersion through the *dispersed belief channel*, is decreasing in the price rigidity while the output gap volatility is increasing in the rigidity due to the *Calvo channel*. The relative weight of price dispersion in the welfare loss function is proportional to the degree of competition in the goods market as the latter amplifies the welfare losses for a given price dispersion. The degree of signal-to-noise ratio and market competition that are required to generate the paradox of price flexibility are satisfied according to empirical estimates of Kalman gain conducted in Coibion and Gorodnichenko (2012, 2015) and empirical estimates of markup, which reflects the degree of competition, studied by De Loecker and Eeckhout (2017).

We extend our static model into a Dynamic Stochastic General Equilibrium model with nominal rigidity and dispersed beliefs as in Nimark (2008). Our finding survives in a dynamic model. Qualitatively, the same results hold independent of whether the underlining shocks that drive the business cycle fluctuations are i) technology shocks, ii) preference shocks.

Our results point to market inefficiency. In our model, from an individual firm's perspective, it would be better off if the firm was allowed to adjust its price more frequently as they would be able to respond to shocks on a timely manner. Therefore, firms would pay for new pricing technologies. However, if all individual firms adopt the new technology to facilitate the price adjustment, the aggregate welfare losses may increase as we discussed above. Thus, the constrained social planner allowing for the presence of dispersed information would not introduce the new technology.

**Literature** This paper is related to two branches of the literature. The theoretical framework is related to the literature that incorporates dispersed beliefs into business cycle models (e.g. Lucas (1972), Woodford (2001), Nimark (2008), Lorenzoni (2009), Angeletos and Jennifer (2009), Hellwig and Venkateswaran (2009), Angeletos and La'O (2011, 2013), Huo and Takayama (2015a,b), Melosi (2016), Angeletos *et al.* (2016), Angeletos and Lian (2018), Huo and Pedroni (2019)).[3] We build on a NK model with dispersed beliefs and derive different components of welfare loss function explicitly.

Our paper contributes to the literature that studies the implications of a reduction in nominal rigidity. The idea that an increase in price flexibility may increase output volatility dates back to

[3]See Mankiw and Reis (2002), Sims (2003), Mackowiak and Wiederholt (2009) for other applications of information frictions for business cycle analysis.

Keynes (1936), was formalized by Long and Summers (1986) and recently it is revisited by Bhattarai *et al.* (2018). Galí (2013) concludes that a reduction in wage rigidity improves welfare only if the central bank reacts to inflation sufficiently aggressive in a closed-economy New Keynesian model featuring both price rigidity and wage rigidity. Following similar reasoning, in an open economy with the fixed exchange rate (Galí and Monacelli (2016)) or a closed economy when the monetary policy is constrained by the Zero Lower Bound (Amano and Gnocchi (2017) and Billi *et al.* (2018)), a labor market reform that results in a more flexible wage is not necessarily welfare improving.

The main contribution of our paper is to combine these two branches of literature, and discuss the role of information frictions or dispersed beliefs in the welfare analysis of nominal rigidity. With this, we introduce a new channel that was previously ignored in the literature: the dispersed belief channel. We show that a more flexible price is welfare detrimental even if i) it does not increase output volatility ii) monetary policy strongly reacts to the inflation.

The remainder of the paper is organized as follows. Section 2 presents a static model and derive the analytical solution to shed light on the role of dispersed information to understand the welfare implications of nominal rigidity. Section 3 builds a dynamic model to quantitative evaluate the welfare gains/losses of the reduction in nominal rigidity. Section 4 concludes.

# 2 Model

## 2.1 Static Model

In this section, we present a static model to explain the key mechanisms.

**Household** There is a representative household with the following period utility function:

$$U = log(C) - L,$$

where $C$ is the amount of goods that the household consume and $L$ is the number of hours that the consumer works.

The household maximizes her utility by choosing the optimal amount of final goods $C$ and total labor $L$ subject to the following budget constraint:

$$PC = WL + T,$$

where, $P$ is the nominal price of goods, $W$ is the nominal wage and $T$ is the firm's profit net of lump-sum taxes paid to the government. The household's optimization problem leads to the following labor supply equation:

$$PC = W \tag{2.1}$$

**Firms** There is a continuum of monopolistic firms producing differentiated intermediate goods using a homogenous production technology. An individual firm $i$'s production function is the following:

$$Y(i) = exp(a)L(i),$$

where $a$ is the log of level productivity that is draw from a normal distribution: $N(0, \sigma_a^2)$. A firm $i$ hires labor $L(i)$ from the representative household. The intermediate goods are aggregated into the final consumption good according to the Dixit-Stiglitz aggregator:

$$Y = \left( \int_0^1 Y(i)^{\frac{\varepsilon - 1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon - 1}},$$

where $\varepsilon > 1$ measures the degree of substitution among varieties and $\frac{\varepsilon}{\varepsilon - 1}$ is the desired markup charged by firms. In the limiting case, $\varepsilon \to \infty$, firms engage into a competitive market. In general, the degree of market competition is increasing in $\varepsilon$.

Consumers's optimal expenditure allocation yields the following demand curve that a firm $i$ faces:

$$Y(i) = \left( \frac{P(i)}{P} \right)^{-\varepsilon} Y, \tag{2.2}$$

where $P \equiv \left( \int_0^1 P(i)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$ denotes the aggregate price.

Firms cannot perfectly observe the technology level $a$ and each firm receives an idiosyncratic signal $s(i)$:

$$s(i) = a + e(i), \tag{2.3}$$

where $e(i)$ is an idiosyncratic noise drawn from a normal distribution $N(0, \sigma_e^2)$. The structure of the model and processes of shocks are common knowledge among all firms.

We assume that only a fraction $1 - \theta$ of firms are allowed to set the price. Each price resetting firm $i$ solves the following maximization problem conditional on its signal:

$$\max_{P(i)} E \left\{ P(i) Y(i) - \frac{W}{Pexp(a)} | s(i) \right\} \tag{2.4}$$

subject (2.2). Combine firms' first order condition with goods and labor market clearing conditions, and linearize it to obtain:

$$p^{\star}(i) = E(p + y - a|s(i)), \tag{2.5}$$

where $p^{\star}(i)$, $p$ and $y$ is the log-deviation from the steady state. $p$ is the aggregate price across all firms defined as $p = \int_i p(i)di$. The remaining fraction $\theta$ of firms keep their prices at initial value zero. Therefore, the aggregate price is:

$$p = (1-\theta)\int p^{\star}(i)di \tag{2.6}$$

**Monetary Policy** The central bank determines an exogenous amount of money in the economy:

$$m = 0. \tag{2.7}$$

In addition, we assume an ad-hoc money demand equation:

$$m - p = y. \tag{2.8}$$

**The Timing of the Model** The model consists of three stages. In stage one, the nature draws a fundamental shock $a$, and each firm $i$ receives a private signal $s_i$ about $a$. Moreover, the structure of the model, including the monetary policy rule and the distribution of shocks are common knowledge across firms. In stage two, each firm $i$ forms a belief and decides a price setting plan. In stage 3, the representative household observes the state of the economy and makes the consumption and labor decision. At the same stage, the goods, labor and money markets clear.

**Model Solution** The solution of the model is characterized by (2.5), (2.6), (2.7), (2.8) and the following Bayes' rule that characterizes firms' beliefs updating:

$$E(a|I_i) = Ks(i), \tag{2.9}$$

with $K \equiv \frac{\sigma_a^2}{\sigma_a^2+\sigma_e^2}$. Solving those equations, we obtain the following aggregate allocations:

$$p = -(1-\theta)Ka, \tag{2.10}$$

$$y = (1-\theta)Ka, \tag{2.11}$$

$$\tilde{y} = (1-\theta)Ka - a, \tag{2.12}$$

where $\tilde{y}$ denotes the output gap.

**Welfare Losses** The object of interest is the representative household's welfare. To this end, we derive the welfare loss function as the second order approximation of the household's utility function:

$$L = \varepsilon var_i p(i) + \tilde{y}^2 \tag{2.13}$$

In the appendix B.1, by combining the welfare loss function with the solution of the model, we show that the ex-ante expected welfare losses can be written as:

$$E(L) = \underbrace{\varepsilon \left\{ \underbrace{(1-\theta)K^2\sigma_e^2}_{\text{Dispersed belief}} + \underbrace{(1-\theta)\theta K^2\sigma_a^2}_{\text{Calvo}} \right\}}_{\text{Price dispersions}} + \underbrace{\big((1-\theta)K-1\big)^2\sigma_a^2}_{\text{Output gap volatility}}. \tag{2.14}$$

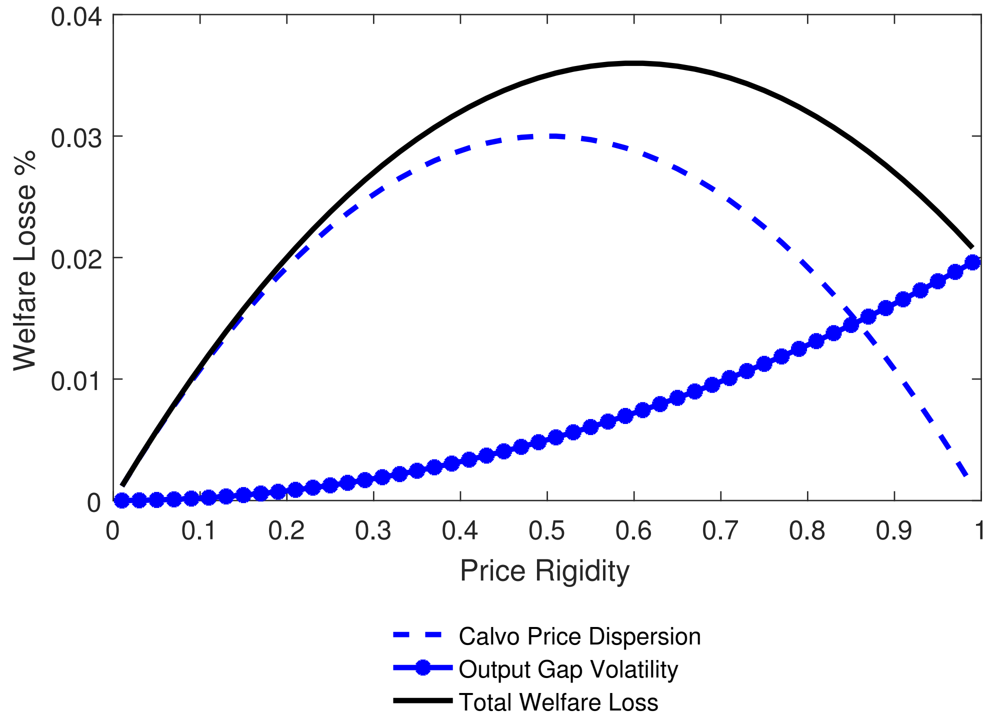
The welfare losses consist of two components: the part that is proportional to the price dispersions and the one that is proportional to the output gap volatility. Such dispersions and volatilities reflect the degree of inefficiency of the economy because goods matter for households' utility symmetrically and production technologies are identical.

The novelty, in a dispersed belief model as compared to the perfect information case, is that there are two channels that drive price dispersions. The first is the standard *Calvo channel* that leads to price dispersions across the group of firms who can reset prices with those who cannot reset prices. The second is the *Dispersed belief channel* that generates price dispersions *within* the group of price resetting firms. Firms form different beliefs about the state of the economy, therefore, in contrast to a standard model with perfect information, price dispersion emerges among those firms who can reset prices.

We are now ready to discuss the welfare implication of an increase in price flexibility. We start with the special case in which there is no information imperfection.

**Proposition 1.** *In the special case in which firms have perfect information, i.e.* $\sigma_\varepsilon = 0$*, the welfare loss as a function of* $\theta$ *is hump-shaped, the economy reaches the maximum welfare loss when* $\theta^{per} = \frac{1}{2}\frac{\varepsilon}{\varepsilon-1}$ *and the welfare loss is minimized if price were flexible.*

**Figure 1: Welfare Losses Decomposition: Perfect Information**




**Note**: This figure plots each component of welfare losses and total welfare losses as a function of price rigidity for the case with perfect information. The elasticity of substitution across goods $\varepsilon$ is calibrated to be 6, which corresponds to a desired markup of 1.2. The volatility of shock $\sigma_a^2$ is calibrated to be 0.02.

With perfect information, the welfare loss function collapses to

$$E(L) = \varepsilon(1-\theta)\theta\sigma_a^2 + \theta^2\sigma_a^2,$$

the Calvo price dispersion component plus the output gap volatility component. It is trivial that the output gap volatility component is strictly increasing in the degree of price rigidity $\theta$. This is the case because the bigger is the nominal rigidity the larger response of the output gap as it is shown in the solution of the model. Price dispersion originating from the Calvo pricing friction is a hump-shaped function of nominal rigidity due to the fact that the Calvo price dispersion peaks if there are as many firms who can reset price as those who cannot, i.e. if $\theta = 0.5$. Therefore, overall, the total welfare losses peak at a value of $\theta$ that is greater than 0.5. The value of the price rigidity that achieves the maximum welfare losses also depends on the $\varepsilon$, the elasticity of substitution across goods as it measures the relative importance of price dispersions against output gap volatility in the welfare loss function.

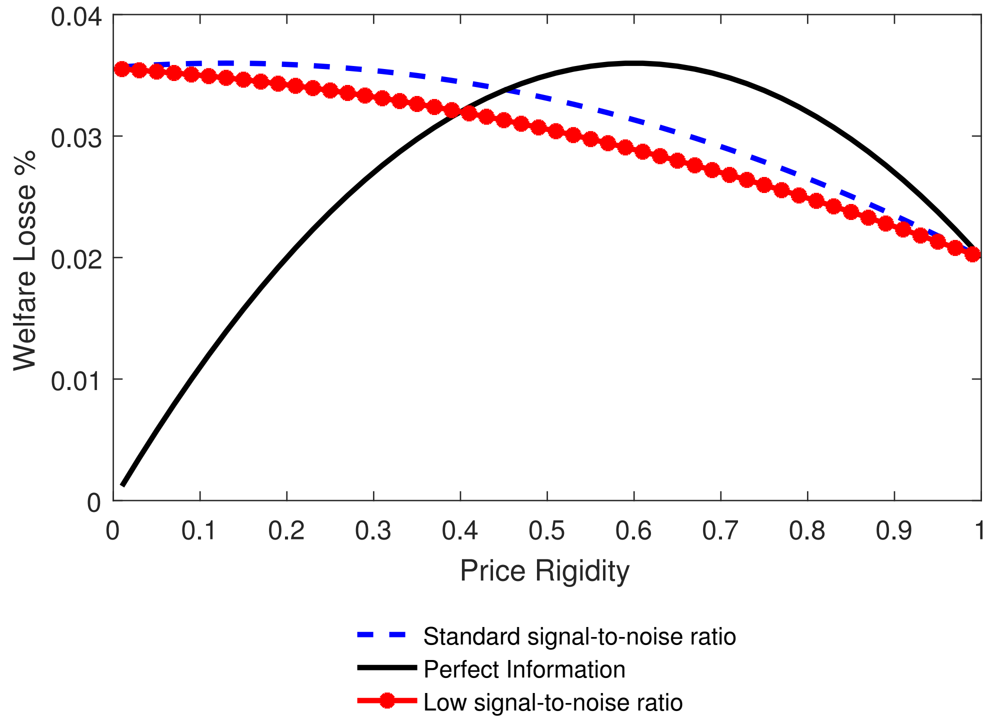
Figure (1) provides a visual representation of those results by plotting the components of welfare loss as a function of price rigidity. The elasticity of substitution across goods $\varepsilon$ is calibrated to be 6, which corresponds to a desired markup of 1.2. The volatility of shock $\sigma_a^2$ is calibrated to be 0.02. Those results suggest that, in a model with perfect information, if the introduction of digital price tags reduces price rigidity $\theta$ towards the left hand side of 0.6, it will be welfare improving.

The presence of information frictions and dispersed beliefs changes the result dramatically:

**Proposition 2.** *In the presence of information frictions and dispersed beliefs, the economy reaches the maximum welfare losses at* $\theta^{imper} = \frac{1}{2}\frac{\varepsilon}{\varepsilon-1} - \frac{\varepsilon-2}{2(\varepsilon-1)}\frac{\sigma_e^2}{\sigma_a^2}$. *Moreover, given a reasonably degree of competition* ($\varepsilon > 2$), $\theta^{imper}$ *is increasing in the signal-to-noise ratio* $\frac{\sigma_a}{\sigma_e}$. *And when the signal-to-noise ratio is sufficiently low, in particular if* $\frac{\sigma_a^2}{\sigma_e^2} < \frac{\varepsilon-2}{\varepsilon}$, *the welfare losses are monotonically decreasing in* $\theta \in [0,1]$.

Figure (3) demonstrates proposition 2 visually. It plots the welfare losses as a function of price rigidity under different scenarios. The solid black line presents the perfect information case. The dashed blue line plots the prediction of the model with dispersed beliefs and the signal-to-noise ratio calibrated to match the empirical Kalman gain of 0.46 estimated by Coibion and Gorodnichenko (2015). The red line with circle shows the result if the signal-to-noise ratio is calibrated to $\frac{\varepsilon-2}{\varepsilon}$, which corresponds to a Kalman gain of $\frac{1}{3}$. Surprisingly, under the reasonable calibration of information frictions (the dashed blue line in Figure 3), in contrast to the perfect information case (solid black line), reducing price rigidity is no longer welfare improving! In fact, the welfare loss is minimized if price were fully rigid.

**Figure 2:** **Welfare Losses: Dispersed Beliefs**

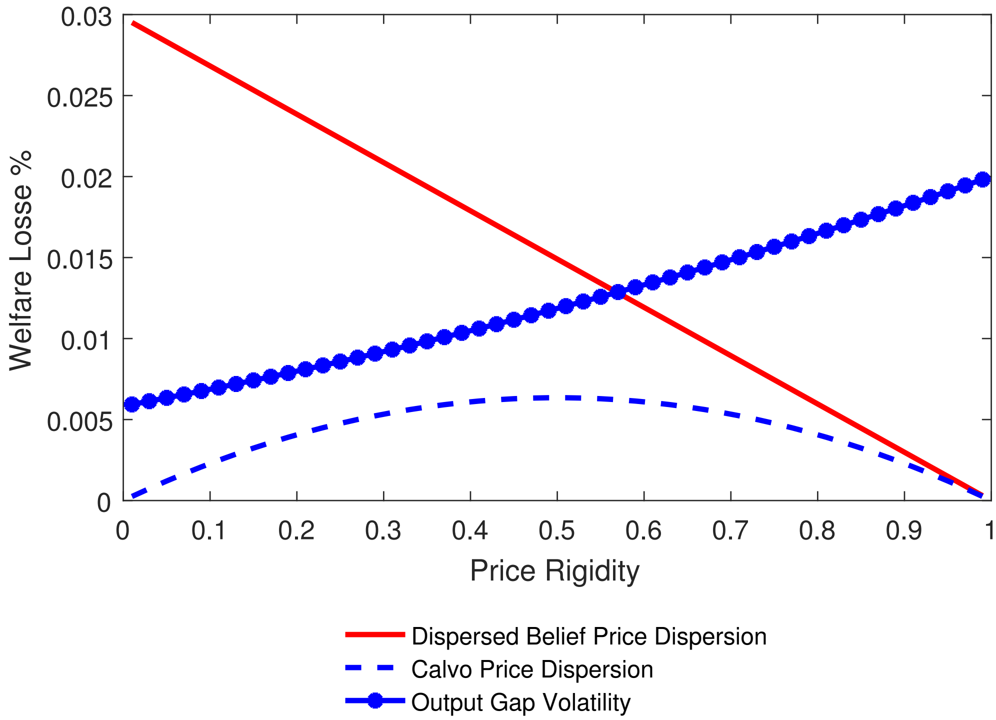



**Note**: This figure plots the welfare losses as a function of price rigidity under different scenarios. The solid black line presents the perfect information case. The dashed blue line plots the prediction of the model with dispersed beliefs with and signal-to-noise ratio calibrated to match a Kalman gain of 0.46. The red line with circle shows the result if the signal-to-noise ratio is calibrated to $\frac{\varepsilon-2}{\varepsilon}$, which corresponds to a Kalman gain of $\frac{1}{3}$. The elasticity of substitution across goods $\varepsilon$ is calibrated to be 6, which corresponds to a desired markup of 1.2. The volatility of shock $\sigma_a^2$ is calibrated to be 0.02.

The above result is entirely driven by the dispersed beliefs channel. With information imperfection and dispersed beliefs, dispersion in prices arises within the group of firms who can reset prices. As one can see in equation (2.14), the dispersed belief component of welfare losses are strictly increasing in the price flexibility $(1-\theta)$. This is verified in Figure (3), which plots each component of welfare losses as a function of price rigidity for the case with dispersed beliefs: the solid red line is strictly decreasing in price rigidity. It is worth to note that if the output gap volatility component (blue line with circles) were sufficiently large, the total welfare losses would be increasing in nominal rigidity. As discussed above, in this model the relative importance of price dispersion v.s output gap volatility depends on $\varepsilon$. This explains why our result requires a $\varepsilon$ that is greater than two. However, in our view this is not a binding condition. There is a consensus in the literature that $\varepsilon$ is calibrated to be between 3 and 11 that correspond to a desired markup in between 1.5 and 1.1. Those numbers are in line with average markup estimated using micro data: see e.g. Berry *et al.* (1995) and Nevo (2001). More recently, De Loecker and Eeckhout (2017)' estimate suggests that the average markup in the U.S. has risen to 1.5, yet it is still far below our condition (a markup of 2).

To conclude this section, in a static model we derive analytically that a reduction in nominal rigidity is not welfare improving when firms possess heterogenous beliefs. In the next section, we extend our baseline static model to a dynamic model and quantitatively evaluate whether our results hold true.

Figure 3: **Welfare Losses Decomposition: Dispersed Beliefs**




**Note**: This figure plots the components of welfare losses as a function of price rigidity for the case with dispersed beliefs. The signal-to-noise ratio calibrated to match a Kalman gain of 0.46. The elasticity of substitution across goods $\varepsilon$ is calibrated to be 6, which corresponds to a desired markup of 1.2. The volatility of shock $\sigma_a^2$ is calibrated to be 0.02.

# 3 Dynamic Model

In the current section, we present a dynamic model to quantitatively evaluate the welfare gains/losses by reducing the price rigidity. The model is based on Nimark (2008), featuring the firms' imperfect knowledge about the state of the economy.

**Household** The representative household maximizes the lifetime utility function:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t exp(d_t) \left\{ \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\nu}}{1+\nu} \right\}$$

subject to the budget constraint:

$$P_t C_t + B_{t+1} = B_t R_t + W_t N_t + T_t.$$

Again $C_t$ denotes the aggregate consumption, $N_t$ is the hours, $W_t$ is the nominal wage, $T_t$ is the firm's profit net of lump-sum taxes paid to the government. $B_{t+1}$ is the quantity of the one-period bond households buy at period t, $R_t$ is the interest rate of the one period bond. $d_t$ is the inter-temporal preference shock that follows an autoregressive process:

$$d_t = \rho_d d_{t-1} + \varepsilon_d$$

Where $\varepsilon_d$ represents an i.i.d. shock with mean zero and constant variance $\sigma_d^2$ that is drawn from a normal distribution, i.e. $\varepsilon_d \sim N(0, \sigma_d)$.

The household has perfect information. Therefore, the consumer's optimality conditions: the Euler equation and the labor supply equation are standard.

**Firm** The final goods producer who operates in a perfect competitive market combines intermediate goods $Y_{i,t}$ for $\forall i \in [0,1]$ according to a CES technology:

$$Y_t = \left( \int_0^1 y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

The profit maximization of the final goods producer leads to the demand function for each intermediate goods $i$:

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t \tag{3.1}$$

for all $i \in [0,1]$.

There is a continuum of monopolistic competitive firms producing the intermediate goods with the following production technology:

$$Y_t(i) = e^{a_t} L_t(i) \tag{3.2}$$

where $e^{a_t}$ is the technology level and $a_t$ evolves according to an autoregressive process $a_t = \rho_a a_{t-1} + \varepsilon_a$, where $\varepsilon_a$ is a Gaussian innovations:$\varepsilon_a \sim N(o, \sigma_a^2)$.

Following Calvo (1983), in each period firms are only allowed to re-optimize their prices with a probability $1-\theta$. Among the faction of firms who can re-optimize they set their prices to maxizmize the expected present discounted value of their future profits:

$$E_t \left\{ \sum_{s=0}^{\infty} (\beta\theta)^s \frac{\lambda_{t+s}}{\lambda_t} \frac{P_t(i)}{P_{t+s}} \left( Y_{t+s}(i) - W_{t+s} N_{t+s}(i) \right) | I_{i,t} \right\}$$

subject to its demand function (3.1) and production function (3.2). $\lambda_t$ is the household marignal utility of nominal income. The remaining $\theta$ fraction of firms, would set the price with $p_t(i) = p_{t-1}(i)$. $I_{i,t}$ is the information set of firm i which we will specify in the following.

**Firm's information set** Firms are subject to information frictions. In particular, each individual firm cannot perfectly observe the state of the economy and receive noisy signals about aggregate shocks.

$$s_{d,i,t} = d_t + \nu_{d,i,t} \tag{3.3}$$

$$s_{a,i,t} = a_t + \nu_{a,i,t} \tag{3.4}$$

where $s_{d,i,t}$ and $s_{a,i,t}$ are imprecise signals about preference shocks, monetary policy shocks and technology shocks respectively that each individual firm receive. $\nu_{d,i,t}$ and $\nu_{a,i,t}$ are the corrsponding noise drawn from the normal distrition $N(0, \sigma^2_{e,d})$ and $N(0, \sigma^2_{e,a})$ respectively. Firms set the price in advance before production takes place based on their information set. Then household supply the labor and consume final product goods. The firms' information set is defined by:

$$I_{i,t} = \left\{ s_{d,j}, s_{a,j}, p_{i,j-1}, y_{i,j-1} : j \leq t \right\}$$

**Monetary Policy** The central bank sets the short term nominal interest rate and follows a Taylor rule:

$$R_t = \left(\frac{\pi_t}{\pi_{ss}}\right)^{\phi_\pi} \left(\frac{y_t}{y_{ss}}\right)^{\phi_y}$$

where $\pi_{ss}$ is the steady state inflation, $y_t$ is the output and $y_{ss}$ is the steady state level of output.

**The Timing of the Model** The timing of model is the same with that we describe in the static model.

**Equilibrium** We log-linearize the model around the perfect information steady state. The equilibrium conditions are:

$$\hat{y}_t = E_t\hat{y}_{t+1} - \frac{1}{\sigma}(\hat{\imath}_t - E_t\hat{\pi}_{t+1}) - (\rho_d - 1)d_t \tag{3.5}$$

$$\hat{\pi}_t = (1-\theta)(1-\beta\theta)\sum_{k=0}^{\infty}(1-\theta)^k \hat{mc}_{t|t}{}^{(k)} + \beta\theta\sum_{k=0}^{\infty}(1-\theta)^k\hat{\pi}^{k+1}_{t+1|t}, \forall k \geq 0 \tag{3.6}$$

$$\hat{mc}_{t|t}{}^{(k)} = (\sigma+\nu)\hat{y}_t^{(k+1)} - (1+\nu)a_t^{(k+1)}, \forall k \geq 0 \tag{3.7}$$

$$\hat{\imath}_t = \phi_\pi\hat{\pi}_t + \phi_y\hat{y}_t \tag{3.8}$$

The equation 3.5 is the standard (log-linearized) Euler equation derives from the household optimization problem. Note that the $E$ is the expectation operator assuming that the household has perfect information. The equation 3.6 is the (log-linearized) New Keynesian phillips curve derived from the firm's price setting equation relying on the assumption that firms cannot perfectly observe the fundamental shocks and set the price based on their information sets as defined in the previous paragraph. $\hat{\pi}^k_{t+1|t}$ is the k-th order expectations about the next period's inflation rate and defined as $\hat{\pi}^k_{t+1|t} \equiv \underbrace{\int E_{i,t} \ldots \int E_{i,t}}_{k} \hat{\pi}_{t+1} di \ldots di$. $E_{i,t}$ denotes the expectation operator conditional on the firm i's information set at time t. $\hat{mc}^{(k)}_{t|t}$ is the k-th order expectations about the marginal cost and defined as $\hat{mc}^k_{t|t} \equiv \underbrace{\int E_{i,t} \ldots \int E_{i,t}}_{k} \hat{mc}_t di \ldots di$. Equation 3.7 defines the marginal cost from the combination of the labor supply equation, the production equation and the definition of marginal cost. Equation 3.8 is the (log-linearized) monetary policy rule.

## 3.1 Solving the model

Following the literature, e.g. Woodford (2001), Nimark (2008) and Melosi (2016), we solve the model by the method of undetermined coefficients. We conjecture that the vector of average expectation of state variables $X_{t|t}^{(0:\infty)}$ follows a VAR(1):

$$\mathbf{X}_{t|t}^{(0:\infty)} = M\mathbf{X}_{t-1|t-1}^{(0:\infty)} + N\varepsilon_t \tag{3.9}$$

where $X_{t|t}^{(0:\infty)} \equiv \left[d_t^{(s)}, a_t^{(s)} : s = 0, 1, ...\infty\right]'$. $\varepsilon_t$ is the vector of exogenous shocks, i.e. $\varepsilon_t \equiv [\varepsilon_{dt}, \varepsilon_{at}]$.

Each individual firm observes a vector of imprecise signal $S_{i,t}$ about the state variable $\mathbf{X}_{t|t}^{(0:\infty)}$:

$$S_{i,t} = D\mathbf{X}_{t|t}^{(0:\infty)} + Qe_{i,t} \tag{3.10}$$

Where $S_{i,t} = [s_{d,i,t}, s_{a,i,t}]$ and $e_{i,t} = [\nu_{d,i,t}, \nu_{a,i,t}]$. Given equation 3.9 and 3.10, each agent forms an estimate about the state variable $X_{t|t}^{(0:\infty)}$. Averaging the individual estimate of the economy, we have a updated law of motion about the state variables.

The vector of endogenous variables $Y_t \equiv [\hat{y}_t, \hat{\pi}_t, \hat{\imath}_t]$ evolves as follows:

$$\mathbf{Y_t} = H\mathbf{X}_{t|t}^{(0:\infty)}$$

We solve for matrix M, N, H such that the equilibrium conditons hold from 3.5 to 3.8 hold. To keep the solution strategy tractable, consistent with the literature, we truncate the infinite order of average expectations up to order $k > 0$. More details are in the Appendix C.

## 3.2 Welfare Losses

Under the assumption of the efficient steady state, the unconditional period welfare losses with information friction, up to a second order approximation are:

$$E(L) = E\left\{\frac{\varepsilon}{\Theta}\frac{1}{1-\beta\theta}(1-\theta)\int_i (p^\star_{i,t} - p^\star_t)^2 di + \frac{\varepsilon}{\Theta}\frac{1}{1-\beta\theta}\frac{\theta}{1-\theta}\pi_t^2 + (\sigma + \frac{\nu+\alpha}{1-\alpha})\tilde{y_t}^2\right\} \quad (3.11)$$

The first term, arising from the dispersed belief in the model, is proportional to the price dispersion within price setting firms. $p^\star_{i,t}$ is the newly set price of firm i in period t and $p^\star_t$ is the average level of the newly set price. The second term captures the welfare losses associated with the inflation volatility that reflects the price dispersion across all firms as if the newly reset prices are restricted to set the average level $p^\star_t$. The last term is proportional to the output gap volatility. In a model with perfect information, the first term will disappear and the welfare loss function will be equivalent to the standard welfare losses in the textbook (e.g. Galí (2015)). The detailed derivations and calculations are in the Appendix B.2.

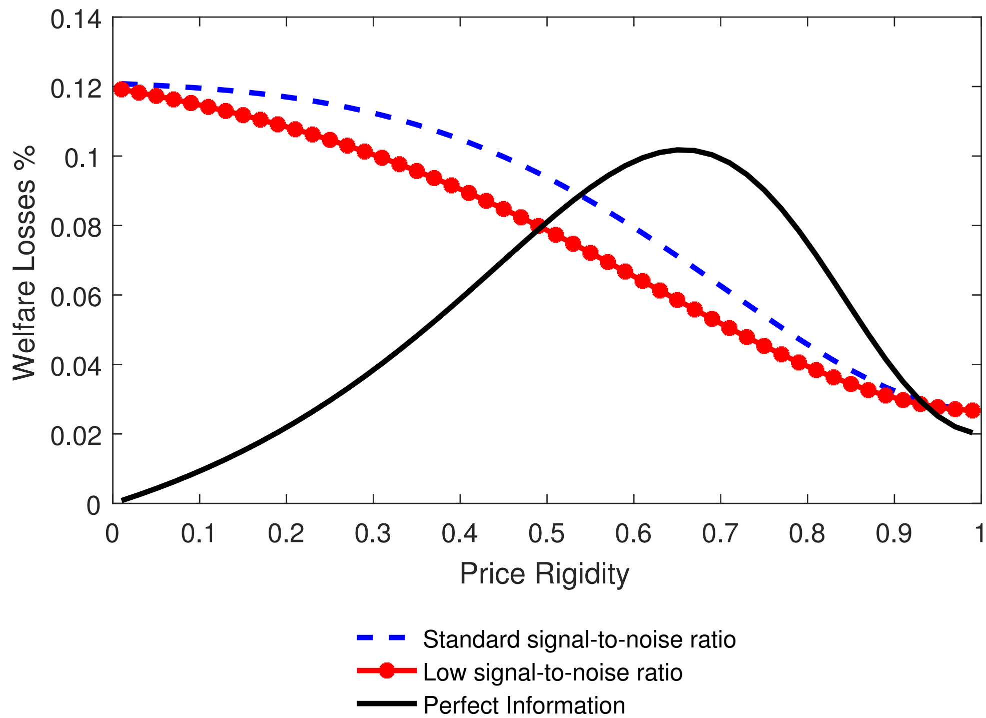
## 3.3 Calibration

The model is calibrated as follows. Each period in the model corresponds to a quarter. The discount factor is 0.99. The inverse Frisch elasticity $\nu$ is 1. The constant relative risk aversion $\sigma$ is 3. The elasticity of substitution across differentiated goods $\varepsilon$ is 6. The Taylor coefficient on inflation is 1.5 and on the output is 0.125. All the volatilities of shocks are set to be 0.02. Consistent with our static model, in our baseline case, the variances of noise are calibrated such that the Kalman gain is 0.46 to be consistent with the estimate of Coibion and Gorodnichenko (2015). We also consider a low signal-to-noise ratio case under which the Kalman gain is 0.3. These parameters are common in the broad business cycle literature.

### 3.4 Results

Figure 4 plots the welfare losses against the price rigidity conditional on technology shocks under different scenarios. The welfare losses in the perfect information model are hump-shaped as we described in the static model. The welfare loss peaks when the price rigidity (probability of remaining the previous price) is around 0.65 which is close to the standard calibration of the price rigidity. According the perfect information model, there is a welfare gain from an increase in the flexibility starting from the rigidity of 0.65. In contrast, based on the impefect information model, there are welfare losses from an increase in price flexibility in both the standard signal-to-noise ratio case (red circled line) and the low signal-to-noise ratio case (blue dashed line). This contrasting result gives rise to quite different policy implications on whether to introduce the new technology to facilitate the price adjustment. If the above perfect information model is the true model, the social planner should encourage firms to undertake the new techonology to faciliate the adujustment cost. As a result, social welfare improves. Constrastly, if the information friction model is right, the social planner would suggest preventing the firms from adopoting the technology.

**Figure 4: Welfare losses conditional on technology shocks: dispersed belief**

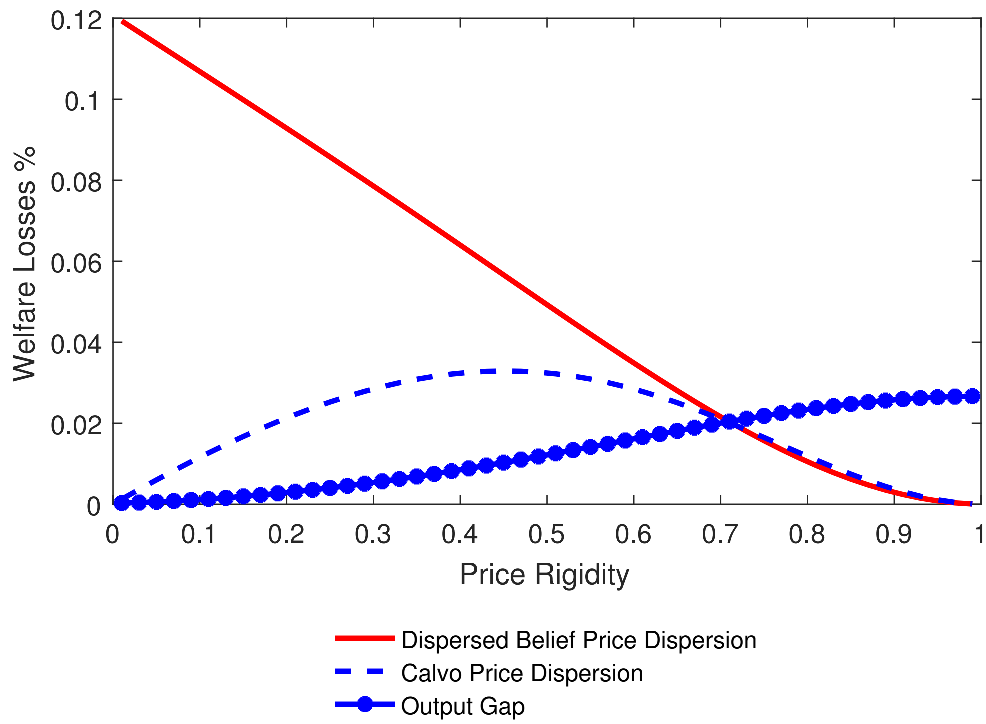



Note: This figure plots the welfare losses as a function of price rigidity under different scenarios. The solid black line presents the perfect information case. The dashed blue line plots the prediction of the model with dispersed beliefs with and signal-to-noise ratio calibrated to match a Kalman gain of 0.46. The red line with circle shows the result if the signal-to-noise ratio is calibrated to match the Kalman gain of $\frac{1}{3}$. The elasticity of substitution across goods $\varepsilon$ is calibrated to be 6, which corresponds to a desired markup of 1.2. The volatility of shock $\sigma_a^2$ is calibrated to be 0.02.

Figure 5 reports each component of the welfare loss function for different values of nominal rigidity. Similar to the static model, the welfare losses associated with the price dispersion within the price resetting firms through the *dispersed belief channel* increases significantly from 0.02% to 0.12% of steady state consumption when the price rigidity is reduced from 0.65 to 0.01. The welfare losses associated with the average inflation volatility and output gap volatility are hump-shaped and increases with the price rigidity respectively, in line with the predictions of the standard textbook model (e.g. Galí (2015)). In our quantitative model, the dispersed belief channel dominates, which generates a welfare loss from an increase in price flexibility.

The welfare losses conditional on the preference shocks share similar patterns and the reasoning. The detailed results are in the Appendix A.

**Figure 5: Welfare losses decomposition conditional on technology shocks**




**Note**: This figure plots the components of welfare loss as a function of price rigidity for the case with dispersed beliefs. The signal-to-noise ratio calibrated to match a Kalman gain of 0.46. The elasticity of substitution across goods $\varepsilon$ is calibrated to be 6, which corresponds to a desired markup of 1.2. The volatility of shock $\sigma_a^2$ is calibrated to be 0.02.

**Discussions** There are two issues worth to discuss in our quantitative model. The first is that our results hold even when the monetary policy response is sufficiently strong. In the model of Galí (2013) and Galí and Monacelli (2016), the result that a reduction in wage rigidity can worsen the welfare occurs when the inflation coefficient in the Taylor rule in the closed-economy model is close to unit or when the exchange rate is fixed in a small open economy model. Both models rely on the mechanism that insufficient response of monetary policy will greatly amplify the employment gap volatility when the wage rigidity decreases. However, our results are based on the increase in the welfare losses associated with the price dispersion within the price resetting firms through dispersed belief channel and the output gap volatility is even decreasing when the price rigidity decreases.

Second, our results point to market inefficiency. In our model, from an individual firm's perspective, it would be better off if the firm was allowed to adjust its price more frequently as they would be able to respond to shocks on a timely manner. Therefore, firms would pay for new pricing technologies. However, if all individual firms adopt the new technology to facilitate the price adjustment, the aggregate welfare losses increase as we discussed above. Thus, the constrained social planner, at the presence of the dispersed information, would not introduce the new technology.

# 4 Conclusion

Is the introduction of digital price tags that may facilitate price adjustment welfare improving? In a New Keynesian model featuring both price rigidity and dispersed information, we show that the answer is no. The dominant underlying mechanism is that a reduction in the price rigidity will amplify the welfare losses associated with the price dispersion within price resetting firms when they have heterogeneous beliefs about the economy. These results add caution to the introduction of the new technology to decrease the price rigidity (e.g. digital price tags).

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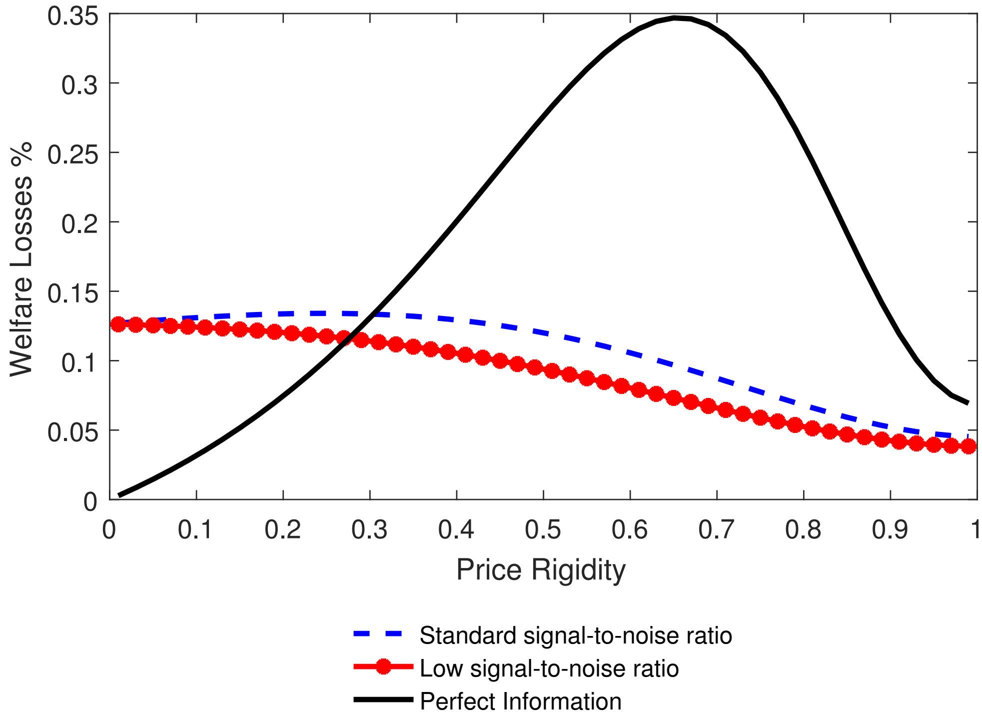
# Appendices

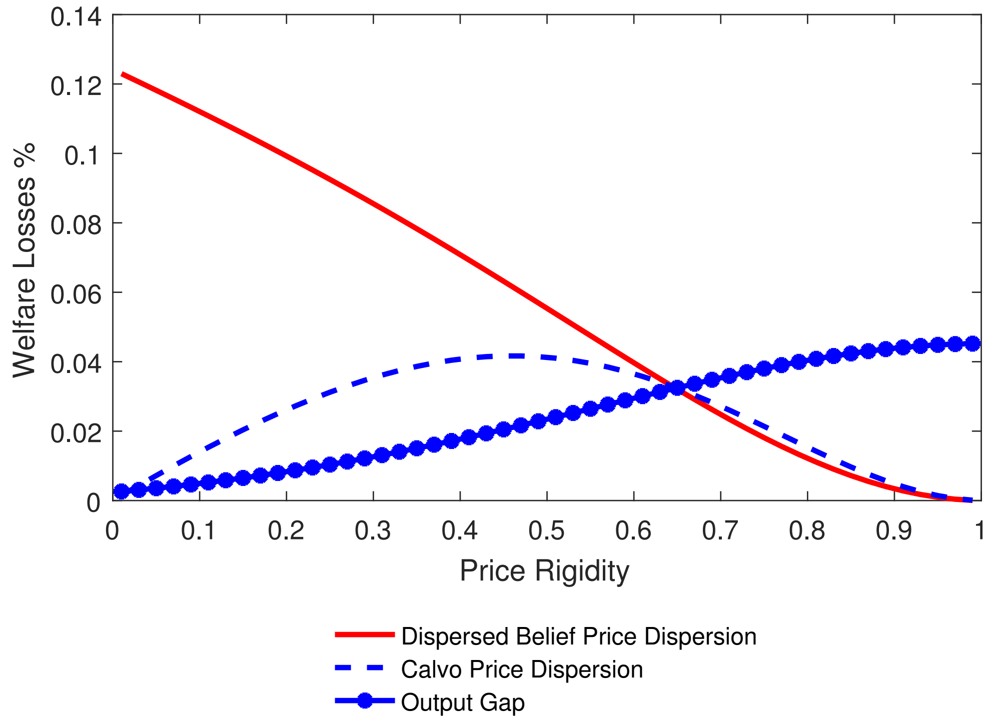
## A Figures

**Figure 6: Welfare losses conditional on preference shocks: dispersed belief**




Note: This figure plots the welfare loss as a function of price rigidity under different scenarios. The solid black line presents the perfect information case. The dashed blue line plots the prediction of the model with dispersed beliefs with and signal-to-noise ratio calibrated to match a Kalman gain of 0.46. The red line with circle shows the result if the signal-to-noise ratio is calibrated to match the Kalman gain of $\frac{1}{3}$. The elasticity of substitution across goods $\varepsilon$ is calibrated to be 6, which corresponds to a desired markup of 1.2. The volatility of shock $\sigma_a^2$ is calibrated to be 0.02.

**Figure 7: Welfare losses decomposition conditional on preference shocks**




**Note**: This figure plots the components of welfare loss as a function of price rigidity for the case with dispersed beliefs. The signal-to-noise ratio calibrated to match a Kalman gain of 0.46. The elasticity of substitution across goods $\varepsilon$ is calibrated to be 6, which corresponds to a desired markup of 1.2. The volatility of shock $\sigma_a^2$ is calibrated to be 0.02.

# B Welfare Losses

## B.1 Static Model

The second-order approximation to the consumer's welfare losses can be written and express as a fraction of steady state consumption as

$$L = \varepsilon var_i p(i) + \tilde{y}^2 \tag{B.1}$$

We first derive the price dipsersion component $var_i p(i)$.

$$\begin{aligned}
var_i p(i) &= \int_i (p(i) - p)^2 di && \text{(B.2)}\\
&= (1-\theta) \int_i (p^\star(i) - p)^2 di + \theta p^2 && \text{(B.3)}\\
&= (1-\theta) \left( \int_i (p^\star(i) - p^\star)^2 di + (p^\star - p)^2 \right) + \theta p^2 && \text{(B.4)}\\
&= (1-\theta) K^2 \int_i e^2(i) di + (1-\theta)\theta p^{\star 2} && \text{(B.5)}\\
&= (1-\theta) K^2 \sigma_e^2 + (1-\theta)\theta K^2 a^2 && \text{(B.6)}
\end{aligned}$$

From the first to the second line, we have used the fact that a fraction of $1-\theta$ of firms set their prices to $p_t^\star(i)$, and a fraction of $\theta$ firms keep the original price 0. From the third line to the fourth line, we have used $p^\star(i) = Ks(i)$, $p^\star = Ka$ and $p = (1-\theta)p^\star$.

$$\tilde{y}^2 = ((1-\theta)K - 1)^2 a^2 \tag{B.7}$$

$$E(L) = \varepsilon \left\{ (1-\theta) K^2 \sigma_e^2 + (1-\theta)\theta K^2 \sigma_a^2 \right\} + ((1-\theta)K - 1)^2 \sigma_a^2. \tag{B.8}$$

## B.2 Dynamic Model

The second-order approximation to the consumer's welfare losses can be written and express as a fraction of steady state consumption as

$$\mathbf{W} = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{\varepsilon}{\Theta} var_i p_t(i) + (\sigma + \frac{\phi + \alpha}{1-\alpha}) \tilde{y}_t{}^2 \right)$$

In the following, we first derive the term $E_0 \sum_{t=0}^{\infty} \beta^t \frac{\varepsilon}{\Theta} var_i p_t(i)$. Second, we decompose it into two components: the price dispersion within the price resetting firms and that across all firms as if the newly price resetting firms are restricted to set the same average price.

**Proposition 3.**

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\varepsilon}{\Theta} var_i p_t(i) = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\varepsilon}{\Theta} \frac{1}{1-\beta\theta} [(1-\theta) \int_i (p^{\star}_{i,t} - p^{\star}_t)^2 di + \frac{\theta}{1-\theta} \pi_t^2]. \tag{B.9}$$

*Proof.* Let $\Delta_t = var_i p_t(i)$, then

$$\begin{aligned}
\Delta_t &= var_i(p_t(i) - p_{t-1}) \\
&= \int_i (p_{i,t} - p_{t-1})^2 di - \left[ \int_i (p_{i,t} - p_{t-1}) di \right]^2 \\
&= \theta \int_i (p_{i,t-1} - p_{t-1})^2 di + (1-\theta) \int_i (p^{\star}_{i,t} - p_{t-1})^2 di - \pi_t^2 \\
&= \theta \Delta_{t-1} + (1-\theta) \int_i (p^{\star}_{i,t} - p_{t-1})^2 di - \pi_t^2 \\
&= \theta \Delta_{t-1} + (1-\theta) \int_i (p^{\star}_{i,t} - p^{\star}_t + p^{\star}_t - p_{t-1})^2 di - \pi_t^2 \\
&= \theta \Delta_{t-1} + (1-\theta) \int_i (p^{\star}_{i,t} - p^{\star}_t)^2 di + (1-\theta) \int_i (p^{\star}_t - p_{t-1})^2 di - \pi_t^2 \\
&= \theta \Delta_{t-1} + (1-\theta) \int_i (p^{\star}_{i,t} - p^{\star}_t)^2 di + \frac{\theta}{1-\theta} \pi_t^2
\end{aligned} \tag{B.10}$$

Finally, from Equation B.10:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\varepsilon}{\Theta} var_i p_t(i) = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\varepsilon}{\Theta} \frac{1}{1-\beta\theta} [(1-\theta) \int_i (p_{i,t}^{\star} - p_t^{\star})^2 di + \frac{\theta}{1-\theta} \pi_t^2] \tag{B.11}$$

$\square$

Therefore, the price dispersion is decomposed into two components: the price dispersion within the price resetting firms and that across all firms as if the newly price resetting firms are restricted to set the same average price. The welfare loss function follows

$$E(L) = E \left\{ \underbrace{\frac{\varepsilon}{\Theta} \frac{1}{1-\beta\theta} (1-\theta) \int_i (p_{i,t}^{\star} - p_t^{\star})^2 di}_{Dispersed\ Belief} + \underbrace{\frac{\varepsilon}{\Theta} \frac{1}{1-\beta\theta} \frac{\theta}{1-\theta} \pi_t^2}_{Calvo} + (\sigma + \frac{\nu + \alpha}{1-\alpha}) \tilde{y}_t^2 \right\} \tag{B.12}$$

After we get the policy function for inflation and output gap, the welfare loss from the variation of inflation and output gap is straightforward. The unique term to this paper is the welfare loss term of the dispersed belief and we didn't postulate the policy function for it directly. Here we illustrate the way to solve for it.

Let $x_t \equiv [d_t; a_t]$ to simplify notation.

We know that

$$p_{i,t}^{*} - p_{t-1} = \frac{1}{1-\theta} \mathbf{b} X_{t|t}^{0}(i) \tag{B.13}$$

$$p_t^{*} - p_{t-1} = \frac{1}{1-\theta} \mathbf{b} X_{t|t}^{0} \tag{B.14}$$

given that $\mathbf{b}$ is the policy function coefficient for inflaion, where $X_{t|t}^{0}(i) = [s_{i,t}\ ; E_i X_{t|t}^{0}] \equiv [s_{i,t}\ ; X_{t|t}^{1}(i)]$

and $X^0_{t|t} = [x_t; EX^0_{t|t}] \equiv [x_t; X^1_{t|t}]$ is the state variable for aggregate endogenous variables in the economy.

Subtracting equation (B.14) from equation (B.14) leads to

$$p^*_{i,t} - p^*_t = \frac{1}{1-\theta}\mathbf{b}(X^0_{t|t}(i) - X^0_{t|t}). \tag{B.15}$$

Then take square of equation (B.15) and sum over individual $i$.

$$\int \left(p^*_{i,t} - p^*_t\right)^2 di = \frac{1}{(1-\theta)^2}\mathbf{b}\int (X^0_{t|t}(i) - X^0_{t|t})(X^0_{t|t}(i) - X^0_{t|t})' di\mathbf{b}'. \tag{B.16}$$

Therefore, we need to find a solution for $\int (X^0_{t|t}(i) - X^0_{t|t})(X^0_{t|t}(i) - X^0_{t|t})' di$.

We have

$$X^1_{t|t}(i) = (M - KDM)X^1_{t-1|t-1}(i) + K[DMX_{t-1} + DN\varepsilon_t + Qe_{i,t}] \tag{B.17}$$

$$X^1_{t|t} = (M - KDM)X^1_{t-1|t-1} + K[DMX_{t-1} + DN\varepsilon_t] \tag{B.18}$$

which will be derived in the next section.

From the above two equations, we have the law of motion for $(X^1_{t|t}(i) - X^1_{t|t})$ following

$$X^1_{t|t}(i) - X^1_{t|t} = (M - KDM)(X^1_{t-1|t-1}(i) - X^1_{t-1|t-1}) + KQe_{i,t} \tag{B.19}$$

We also know that

$$s_{i,t} - x_t = Qe_{i,t} \tag{B.20}$$

Combine the above two equations, we obtain

$$X^0_{t|t}(i) - X^0_{t|t} = \bar{M}(X^0_{t-1|t-1}(i) - X^0_{t-1|t-1}) + \bar{N}e_{i,t}. \tag{B.21}$$

where

$$\bar{M} = \begin{bmatrix} 0 & 0 \\ 0 & M - KDM \end{bmatrix}$$

$$\bar{N} = \begin{bmatrix} Q \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ KQ \end{bmatrix}.$$

Utilizing equation (B.21), we can obtain a solution for $\int (X^0_{t|t}(i) - X^0_{t|t})(X^0_{t|t}(i) - X^0_{t|t})' di$.

# C Solution Method

The observation equation is

$$Z_t(i) = L\mathbf{X}_t + Q\nu_t(i), \tag{C.1}$$

where $Z_t(i) = [s_{dt}(i)\ s_{\eta t}(i)\ s_{at}(i)]'$ ,$\nu_t(i) = [\nu_{dt}(i)\ \nu_{\eta t}(i)\ \nu_{at}(i)]'$and $Q$ is

$$Q \equiv \begin{bmatrix} \sigma_{\nu_d} & 0 & 0 \\ 0 & \sigma_{\nu_a} & 0 \\ 0 & 0 & \sigma_{\nu_\eta} \end{bmatrix}.$$

As is decribed earlier, the law of motion for state is

$$\mathbf{X}_t = M\mathbf{X}_{t-1} + N\varepsilon_t \tag{C.2}$$

Define variance matrix for the noise and for the shock by $\Sigma_{\nu\nu}$ and $\Sigma_{\varepsilon\varepsilon}$ respectively satisfying

$$\Sigma_{\nu\nu} \equiv QQ^{'}$$

$$\Sigma_{\varepsilon\varepsilon} \equiv N \begin{bmatrix} \sigma_{\varepsilon_d}^2 & 0 & 0 \\ 0 & \sigma_{\varepsilon_a}^2 & 0 \\ 0 & 0 & \sigma_{\varepsilon_\eta}^2 \end{bmatrix} N'.$$

Equations (C.1) and (C.2) consist of linear space which calls for the Kalman filter.

The steady state Kalman filter gives

$$\begin{aligned} \mathbf{X}_{t|t}^{(1)}(i) &= (M - KLM)\mathbf{X}_{t-1|t-1}^{(1)}(i) + KZ_t(i), && \text{(C.3)} \\ &= (M - KLM)\mathbf{X}_{t-1|t-1}^{(1)}(i) + K[LM\mathbf{X}_{t-1} + LN\varepsilon_t + Q\mathbf{e}_t(i)], && \text{(C.4)} \end{aligned}$$

where

$$K_l = PL'(LPL' + \Sigma_{\nu\nu})^{-1},$$

with

$$P = M(P - PL'(LPL' + \Sigma_{\nu\nu})^{-1}LP)M^{'} + \Sigma_{\varepsilon\varepsilon}.$$

Take average over individual expectation, we have

$$\begin{aligned} \mathbf{X}_{t|t}^{(1)} &= (M - KLM)\mathbf{X}_{t-1|t-1}^{(1)} + KZ_t && \text{(C.5)} \\ &= (M - KLM)\mathbf{X}_{t-1|t-1}^{(1)} + K[LM\mathbf{X}_{t-1} + LN\varepsilon_t] && \text{(C.6)} \end{aligned}$$

Recall that the shocks evolve as

$$\begin{bmatrix} d_t \\ a_t \end{bmatrix} = \rho \begin{bmatrix} d_{t-1} \\ a_{t-1} \end{bmatrix} + R\varepsilon_t, \tag{C.7}$$

where $\rho \equiv \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_a \end{bmatrix}$ and $R \equiv \begin{bmatrix} \sigma_{\varepsilon_d} & 0 \\ 0 & \sigma_{\varepsilon_\eta} \end{bmatrix}$.

Equation (C.5) and equation (C.7) can fully characterize the matrices $M$ and $N$ as follows

$$M = \begin{bmatrix} \rho & \mathbf{0}_{3\times 3J} \\ \mathbf{0}_{3J\times 3} & (M - KLM)\,|_{(1:3J,1:3J)} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{3\times 3(J+1)} \\ KLM\,|_{(1:3J,1:3(J+1))} \end{bmatrix}, \tag{C.8}$$

and

$$N = \begin{bmatrix} R \\ \mathbf{0}_{3J\times 3} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{3\times 3} \\ KLN\,|_{(1:3J,1:3)} \end{bmatrix}. \tag{C.9}$$

When firms don't hvae endogenous signals, the problem of solving the law of motion for state and policy function can be seperated. Equation (C.8) and (C.9) can be used to iterate to obtain a fixed point for $M$ and $N$.

Given the law of motion for state, we can compute for policy functions. Let $\hat{y}_t = \mathbf{a}\mathbf{X}_{t|t}^{(0:J)}$ and $\hat{\pi}_t = \mathbf{b}\mathbf{X}_{t|t}^{(0:J)}$ as output and inflation are a linear function of hierachy of state variables.

Let $\mathbf{s}_t \equiv [\hat{y}_t\ \hat{\pi}_t]'$ and $\mathbf{v}_0 \equiv \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix}$. And define matrix $T^{(s)}$ as follows:

$$T^{(s)} = \begin{bmatrix} \mathbf{0}_{3(J-s+1)\times 3s} & I_{3(J-s+1)} \\ \mathbf{0}_{3s\times 3s} & \mathbf{0}_{3s\times 3(J-s+1)} \end{bmatrix}.$$

Then we have the following important results for higher order expectation of endogenous variables

$$
\begin{aligned}
\mathbf{s}_{t|t}^{(s)} &= \mathbf{v}_0 T^{(s)} X_{t|t}^{(0:J)}, \\
\mathbf{s}_{t+h|t}^{(s)} &= \mathbf{v}_0 M^h T^{(s)} X_{t|t}^{(0:J)}.
\end{aligned}
$$

for any $0 \leq s \leq J$.

Recall that IS curve and NKPC after the substitution of policy function are

$$
\begin{aligned}
\mathbf{a}X_{t|t}^{(0:J)} &= \mathbf{a}MT^{(1)}X_{t|t}^{(0:J)} - \frac{1}{\sigma}(\phi_\pi \mathbf{b}X_{t|t}^{(0:J)} + \phi_y \mathbf{a}X_{t|t}^{(0:J)} + \chi X_{t|t}^{(0:J)} - \mathbf{b}MT^{(1)}X_{t|t}^{(0:J)}) \\
&\quad +(\rho_d - 1)\gamma X_{t|t}^{(0:J)}, \qquad \text{(C.10)} \\
\mathbf{b}X_{t|t}^{(0:J)} &= (1-\theta)(1-\beta\theta)\sum_{j=1}^{J}(1-\theta)^{j-1}((\sigma+\nu)\mathbf{a}T^{(j)}X_{t|t}^{(0:J)} - (1+\nu)GX_{t|t}^{(0:J)}) \\
&\quad +\beta\theta\sum_{j=1}^{J}(1-\theta)^{j-1}\mathbf{b}MT^{(j)}X_{t|t}^{(0:J)}. \qquad \text{(C.11)}
\end{aligned}
$$

which lead to

$$
\begin{aligned}
\mathbf{a} &= \mathbf{a}MT^{(1)} - \frac{1}{\sigma}(\phi_\pi \mathbf{b} + \phi_y \mathbf{a} + \chi - \mathbf{b}MT^{(1)}) + (\rho_d - 1)\tau, \qquad \text{(C.12)} \\
\mathbf{b} &= (1-\theta)(1-\beta\theta)\sum_{j=1}^{J}(1-\theta)^{j-1}((\sigma+\nu)\mathbf{a}T^{(j)} - (1+\nu)G) \\
&\quad +\beta\theta\sum_{j=1}^{J}(1-\theta)^{j-1}\mathbf{b}MT^{(j)}, \qquad \text{(C.13)}
\end{aligned}
$$

where

$$
\begin{aligned}
\tau &\equiv [1\ 0\ 0\ \mathbf{0}_{1\times 3J}], \\
G &\equiv [\mathbf{0}_{1\times 3s}\ 0\ 1\ 0\ \mathbf{0}_{1\times 3(J-s)}], \\
\chi &\equiv [0\ 0\ 1\ \mathbf{0}_{1\times 3J}].
\end{aligned}
$$

Equation (C.12) and (C.13) characterize the restrictions on the postulated policy functions. Iterating over **a** and **b** according to equation (C.12) and (C.13) until they converge gives rise to the solution for the policy functions.